Lab 7 — MATH 1586 Spring 2018

Recall: this site plots surfaces nicely:
http://web.monroecc.edu/manila/webfiles/pseeburger/CalcPlot3D/
Also, it is helpful to use WolframAlpha to compute integrals. You may also of course use Mathcad to do it.
This lab concerns some applications of line integrals. Recall that if C is a curve parametrized by $\vec{r}(t)$ for $a \leq t \leq b$, then

$$\int_C f(x,y,z)\mathrm{d}s = \int_a^b f(x(t),y(t))\|\vec{r}\,'(t)\|\mathrm{d}t$$

and if $\vec{F}(x,y,z)$ is a vector field, then

$$\int_C \vec{F}\cdot\mathrm{d}\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\mathrm{d}t.$$

1.) Do the following parts.

a.) Plot the surface $z = 2 - x^2 - y^2$ and plot where the curve $\vec{r}(t) = \left\langle \cos^3(t), \sin^3(t)\right\rangle$ for $0 \leq t \leq \dfrac{\pi}{2}$ lies on the surface. (i.e. plot $\left\langle \cos^3(t), \sin^3(t), 2 - \cos^6(t) - \sin^6(t)\right\rangle$ for $0 \leq t \leq \dfrac{\pi}{2}$ using the "Add to graph: Space curve" drop down menu).

b.) Include a picture of your part a in your lab, making sure to also show the curve lying on the surface.

c.) Find $\vec{r}\,'(t)$ by using WolframAlpha.

d.) Set up the appropriate integral to compute $\displaystyle\int_C 2 - x^2 - y^2\mathrm{d}s$ and use WolframAlpha to compute its value.

2.) Do the following parts.

a.) Plot the vector field $\vec{F}(x,y,z) = \langle \sin(y), x\cos(y), xy\rangle$ (useful: you'll find the thing for it in the dropdown menu; click checkbox "use fixed length for all vectors", and mode "cylindrical array" with "in each circle" set to 8, "# circle" set to 8, and "along z-axis" set to 10). Also plot the space curve $\vec{r}(t) = \left\langle \dfrac{\sin(t)}{(t+1)^2}, \dfrac{\sin(t)}{t+1}, t\sin(3t)\right\rangle$.

b.) Include a picture of what you plotted above.

c.) Use WolframAlpha to compute $\vec{r}\,'(t)$.

d.) Write down the integral you would have to solve to compute $\displaystyle\int_C \vec{F}\cdot\mathrm{d}\vec{r}$ and use WolframAlpha to compute its value.