Lab 6 — MATH 1586 Spring 2018

Recall: this site plots surfaces nicely:
http://web.monroecc.edu/manila/webfiles/pseeburger/CalcPlot3D/
Also, it is helpful to use WolframAlpha to compute integrals. You may also of course use Mathcad to do it.

This lab concerns the application of double integrals to probability theory. Let D be a region in the plane (called the domain) and let $f(x, y)$ be a function which describes a surface above D. We say that f is a probability density function if the following two properties hold:

i.) for every point (x, y) in D, $f(x, y) \geq 0$, and

ii.) $\iint_D f(x, y) \mathrm{d}A = 1$.

A set E inside of D is called an event. It is often asked in probability theory to find the "probability of an event" occurring. This is what we will calculate in this lab.

1. Consider the "normal" bivariate probability density function "with mean 0 and standard deviation 1" given by $f(x, y) = \dfrac{1}{2\pi} e^{-\frac{x^2+y^2}{2}}$ with domain $D = \mathbb{R}^2$ (the whole plane). Consider the event defined by $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$. Attach a picture of the distribution over this region and also find the probability of this event.

2. Use the distribution in problem 1 to find the probability that $x \leq y$ in the region E. To do this, draw the curve $y = x$ in the region with $-2 \leq x \leq 2$ and $-2 \leq y \leq 2$. Shade the half of this drawing that agrees with "$x \leq y$" in. Set up the integral over this region (it should be triangular; draw $y = x$ and then figure out which of the two halves obeys $x \leq y$) and then use that to find the probability.

3. Consider the domain D determined by $-1 \leq x \leq 1$ and $-1 \leq y \leq 1$ and the distribution $f(x, y) = \dfrac{3}{8}(x^2 + y^2)$. Plot a picture of this distribution and find the probability that $y \leq x$ in the region.