Lab 5 — MATH 1586 Spring 2018

We will use this webpage today:
http://web.monroecc.edu/manila/webfiles/pseeburger/CalcPlot3D/
We will look at surfaces and the gradient vector field in this lab. Recall the directional derivative in the direction of the **unit** vector $\vec{u} = \langle a, b \rangle$:

$$D_{\vec{u}} f(x_0, y_0) = \vec{u} \cdot \nabla f(x_0, y_0) = a \frac{\partial f}{\partial x}(x_0, y_0) + b \frac{\partial f}{\partial y}(x_0, y_0).$$

Recall the following theorem from class:
Theorem: The maximum value of $D_u f\,(a, b)$ is $\|\nabla f\,(a, b)\|$ and it occurs in the same direction as the gradient vector $\nabla f\,(a, b)$.

1. Let $f(x, y) = 2 - x^2 - y^2$. Plot the surface $z = f(x, y)$. We saw in class that the gradient of this function is $\nabla f = \langle -2x, -2y \rangle$. Plot the vector field as well. Notice that the vector arrows always point in the direction of "steepest ascent"! Take a screenshot of this final image and attach it to your lab (in Windows: "Print Screen" key copies the screen to clipboard, then open mspaint and paste it there to get the picture).

2. An electric potential V of a charged plate is given by $V(x, y) = x^2 y - x y^2$.

   a.) Find the rate of change of the potential at $(1, 0)$ in the direction of $\vec{v} = \langle -1, -1 \rangle$.

   b.) In which direction does V change most rapidly?

   c.) What is the maximum rate of change at $(1, 1)$?

   d.) Draw V and ∇V in CalcPlot3D. Screenshot and observe the results you found above.

3. The temperature on a metal plate at point (x_0, y_0) is given by $T(x, y) = 2e^{-x^2 - 3y^2}$.

   a.) Find the rate of change of the temperature of the plate at the point $(1, 1)$ in the direction of the vector $\langle -1, -1 \rangle$.

   b.) In which direction does the temperature change the fastest at $(1, 1)$?

   c.) What is the maximum rate of temperature change at $(1, 1)$?

   d.) Draw V and ∇V in CalcPlot3D. Screenshot and observe the results you found above.