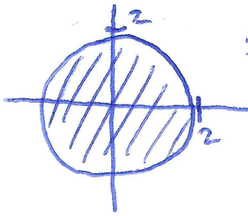
HW9 MATH 1586

① region:



As polar region

$0 \le r \le 2$

$0 \le \theta \le 2\pi$

So,

$$\iint_R \underbrace{x^2+y^2}_{=r} \underbrace{dA}_{=r\,dr\,d\theta} = \int_0^{2\pi}\int_0^2 r^2 \; r\,dr\,d\theta$$

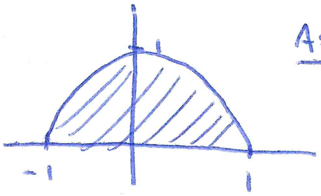
$$= \int_0^{2\pi}\int_0^2 r^3\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{r^4}{4}\Big|_{r=0}^{r=2} d\theta$$

$$= \int_0^{2\pi} \frac{16}{4}\,d\theta$$

$$= \frac{32\pi}{4} = \frac{16\pi}{2} = 8\pi$$

② region:



As polar region

$0 \le r \le 1$

$0 \le \theta \le \pi$

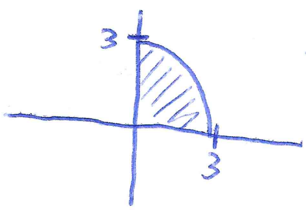
So,

$$\iint_R \sqrt{x^2+y^2}\,dA = \int_0^{\pi}\int_0^1 \underbrace{\sqrt{r^2}}_{=r}\; r\,dr\,d\theta$$

$$= \int_0^{\pi}\int_0^1 r^3\,dr\,d\theta$$

$$= \int_0^{\pi} \frac{r^4}{4}\Big|_0^1 d\theta = \int_0^{\pi} \frac{1}{4}\,d\theta = \frac{2\pi}{4} = \frac{\pi}{2}$$

③ region



As polar region

$0 \le r \le 3$

$0 \le \theta \le \frac{\pi}{2}$

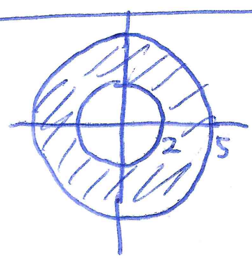
So,

$$\iint_R \sqrt{x^2+y^2}\,dA = \int_0^{\pi/2}\int_0^3 \sqrt{r^2}\; r\,dr\,d\theta$$

$$= \int_0^{\pi/2}\int_0^3 r^2\,dr\,d\theta = \int_0^{\pi/2} \frac{r^3}{3}\Big|_0^3 d\theta$$

$$= \int_0^{2\pi} \underbrace{\frac{27}{3}}_{=9} d\theta = 18\pi$$

④ region



As polar

$2 \le r \le 5$

$0 \le \theta \le 2\pi$

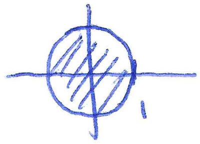
So,

$$\iint_R x^2+y^2\,dA = \int_0^{2\pi}\int_2^5 \underbrace{r^2 \; r}_{=r^3}\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{r^4}{4}\Big|_2^5 d\theta = \int_0^{2\pi} \frac{5^4}{4} - \frac{2^4}{4}\,d\theta$$

$$= 2\pi\left(\frac{5^4}{4} - 4\right)$$

(5)



$0 \le r \le 1$

$0 \le \theta \le 2\pi$

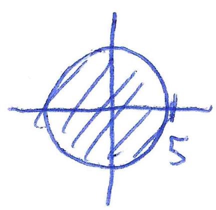
$$\iint_R (x^2+y^2)^2 + (x^2+y^2)\,dA = \int_0^{2\pi}\int_0^1 \underbrace{\left((r^2)^2+r^2\right)}_{r^5+r^3} r\,dr\,d\theta$$

$$= \int_0^{2\pi} \frac{r^6}{6} + \frac{r^4}{4}\Big|_0^1 \,d\theta$$

$$= \int_0^{2\pi} \frac{1}{6}+\frac{1}{4}\,d\theta$$

$$= 2\pi\left(\frac{1}{6}+\frac{1}{4}\right)$$

(6)



$0 \le r \le 5$

$0 \le \theta \le 2\pi$

$$\iint_R e^{x^2+y^2}\,dA = \int_0^{2\pi}\int_0^5 e^{r^2}\, r\,dr\,d\theta$$

$u = r^2$

$\frac{1}{2}du = r\,dr$

$r=0 \rightarrow u=0$

$r=5 \rightarrow u=25$

$$= \int_0^{2\pi}\int_0^{25} e^u\,du\,d\theta$$

$$= \int_0^{2\pi} e^u\Big|_0^{25}\,d\theta$$

$$= \int_0^{2\pi} (e^{25}-1)\,d\theta$$

$$= 2\pi(e^{25}-1)$$