Homework 7 — MATH 1586 Spring 2018

Recall that the tangent plane to a surface $z = f(x, y)$ above the point (a, b) is given by

$$z = f(a,b) + \frac{\partial f}{\partial x}(a,b)(x-a) + \frac{\partial f}{\partial y}(a,b)(y-b).$$

Recall that the gradient of a function $f(x, y)$ or a function $f(x, y, z)$ is given by

$$\nabla f(x,y) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

Recall the magnitude of a vector $\langle a, b\rangle$ is its length, i.e. $\|\langle a, b\rangle\| = \sqrt{a^2 + b^2}$. We say that a vector is a unit vector if it is length 1. You can make any vector $\vec{v}$ into a unit vector $\vec{u}$ pointing in the same direction as $\vec{v}$ by dividing by its magnitude: $\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}$.

Let $\vec{u}$ be a unit vector and let $f(x, y)$ be a function. We say that the directional derivative of f in the direction $\vec{u} = \langle a, b\rangle$ at (x_0, y_0) is given by

$$D_{\vec{u}} f(x_0, y_0) = \vec{u} \cdot \nabla f(x_0, y_0) = a f_x(x_0, y_0) + b f_y(x_0, y_0).$$

Also recall the following theorem:

Theorem: Suppose that $f(\vec{x})$ where $\vec{x} = \langle x_1, \ldots, x_n\rangle$ is a partial differentiable function of all variables $x_1, x_2, \ldots, x_n$. The maximum value of $D_u f(\vec{x})$ is $\|\nabla f(\vec{x})\|$ and it occurs in the same direction as the gradient vector $\nabla f(\vec{x})$. ■

Recall that to compute "iterated integrals" like

$$\int_a^b \int_c^d f(x,y)\mathrm{d}x\mathrm{d}y$$

is just to work "inside-out". The region of integration of such an integral is the square whose top and bottom sides are above $[a, b]$ on the x-axis and whose left and right sides are next to $[c, d]$ on the y-axis.

1. Find an equation for the tangent plane to $f(x, y) = x^2y^3 + xy^2$ above the point $(2, 3)$.

2. Calculate the gradient of $f(x, y) = x^2e^{xy} + y \log(x)$.

3. Find a unit vector that points in the same direction as the vector $\vec{v} = \langle 2, 3\rangle$.

4. Find the directional derivative of $f(x, y) = xy^2 + e^{xy}$ in the direction $\left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \right\rangle$ (note: this vector is already a unit vector) at $(x_0, y_0) = (1, 2)$.

5. Find the directional derivative of $f(x, y) = e^x \log(y) + x^2y$ in the direction of $\langle -1, 2\rangle$ at $(x_0, y_0) = (2, 2)$.

6. Calculate
$$\int_0^1 \int_2^3 x^2y + xy^2\mathrm{d}x\mathrm{d}y$$
and draw the region of integration.