HW 6

① $f(1,2,3) = (1^2)(2) + (1)(2)(3) + 3^2$

$= 2 + 6 + 9$

$= 17$

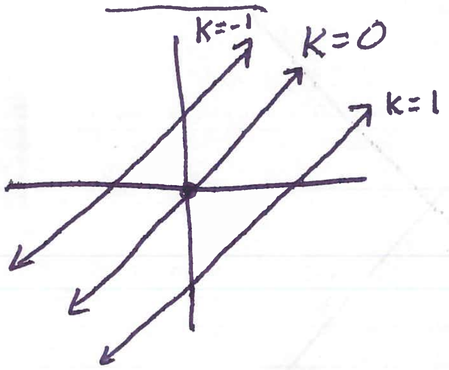
② $\underline{k=-1}$ $\underline{k=0}$ $\underline{k=1}$

$-1 = 4x - y$ $0 = 4x - y$ $1 = 4x - y$

$\Downarrow$ $\Downarrow$ $\Downarrow$

$y = 4x + 1$ $y = 4x$ $y = 4x - 1$

Draw



③ $\underline{k=0}$

$0 = \sqrt{9-x^2-y^2}$

$\downarrow$ square both sides

$0 = 9 - x^2 - y^2$

$\downarrow$

circle of radius 3 $\longrightarrow$ $x^2 + y^2 = 9$

$\underline{k=2}$

$2 = \sqrt{9-x^2-y^2}$

$\downarrow$

$4 = 9 - x^2 - y^2$

$\downarrow$

$x^2 + y^2 = 5$

circle, radius $\sqrt{5}$

$\underline{k=3}$

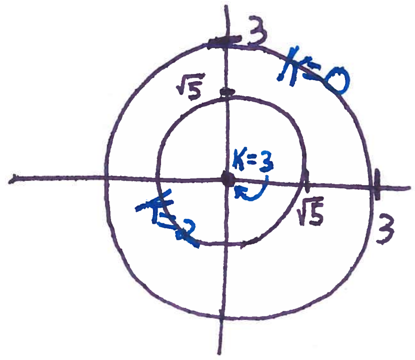
$3 = \sqrt{9-x^2-y^2}$

$9 = 9 - x^2 - y^2$

$\downarrow$

$x^2 + y^2 = 0$

circle of radius 0... a point!

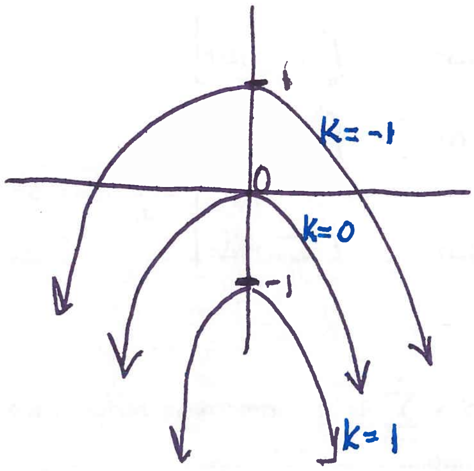


(4) $\underline{K=-1}$ $\underline{K=0}$ $\underline{K=1}$

$-1=-2x^2-y$ $\downarrow$ $y=-2x^2+1$

$0=-2x^2-y$ $\downarrow$ $y=-2x^2$

$1=-2x^2-y$ $\downarrow$ $y=-2x^2-1$



(5) $\underline{\text{Path 1}}$: x-axis $\uparrow$ "(x,0)"

Along this path, limit becomes

$$\lim_{(x,0)\to(0,0)} \frac{x(0)}{\cancel{x}^2+3(0^2)} = \lim_{x\to 0} 0 = 0$$

$\underline{\text{Path 2}}$: line $y=x$ $\nwarrow$ "(x,x)"

Along this path, limit becomes

$$\lim_{(x,x)\to(0,0)} \frac{x(x)}{x^2+3x^2} = \lim_{x\to 0} \frac{x^2}{4x^2} = \lim_{x\to 0} \frac{1}{4} = \frac{1}{4}$$

different values $\Rightarrow$ limit DNE

⑥ Path 1: x-axis
$\nwarrow (x,0)$

Limit becomes

$$\lim_{(x,0)\to(0,0)} \frac{3x^2+4(0)}{3x+4(0)} = \lim_{x\to 0} \frac{3x^2}{x} = \lim_{x\to 0} 3x = 0$$

different numbers
$\Rightarrow$ limit DNE

Path 2: y-axis
$\nwarrow$ "$(0,y)$"

Limit becomes

$$\lim_{(0,y)\to(0,0)} \frac{3(0^2)+4y}{3(0)+4y} = \lim_{y\to 0} \frac{4y}{4y} = \lim_{y\to 0} 1 = 1$$

⑦ $$\lim_{(x,y)\to(3,4)} e^{xy}(x^2-3y) = e^{3(4)}(3^2-3(4))$$
$$= e^{12}(9-12)$$
$$= -3e^{12}$$

⑧ $$\frac{\partial f}{\partial x} = 6xy + 0 = 6xy \qquad \frac{\partial f}{\partial y} = 3x^2 + 15y^2$$

⑨ $$\frac{\partial f}{\partial x} \underset{\substack{\uparrow\\ \text{prod}\\ \text{RULE}}}{=} \underbrace{\left(\frac{\partial}{\partial x} e^{xy}\right)}_{= e^{xy}\frac{\partial}{\partial x}xy = ye^{xy}}\left(z^2x + z\ln(y) + x^2yz\right) + e^{xy}\left(\frac{\partial}{\partial x}\left[z^2x + z\ln(y) + x^2yz\right]\right)$$

$$= ye^{xy}\left(z^2x + z\ln(y) + x^2yz\right) + e^{xy}\left(z^2 + 0 + 2xyz\right)$$

Similarly,
$$\frac{\partial f}{\partial y} = xe^{xy}\left(z^2x + z\ln(y) + x^2yz\right) + e^{xy}\left(\frac{z}{y} + x^2z\right), \text{ and}$$

$$\frac{\partial f}{\partial z} = e^{xy}\left(2zx + \ln(y) + x^2y\right)$$