HW5

1) $\langle 3,7,10\rangle + 2\langle 3,1,-1\rangle = \langle 3,7,10\rangle + \langle 6,2,-2\rangle$

$= \langle 9,9,8\rangle$

2) $\langle 1,6,2\rangle \cdot \langle 7,1,3\rangle = 1\cdot 7 + 6\cdot 1 + 2\cdot 3$

$= 7+6+6$

$= 19$

3) $\det\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix} = 3(-1) - 2(1) = -3-2 = -5$

4) $\langle 1,0,1\rangle \times \langle 2,1,3\rangle = \det\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

$= \vec{i}\det\begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix} - \vec{j}\det\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} + \vec{k}\det\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$= \vec{i}(0-1) - \vec{j}(3-2) + \vec{k}(1-0)$

$= \langle -1,0,0\rangle + \langle 0,-1,0\rangle + \langle 0,0,1\rangle$

$= \langle -1,-1,1\rangle$

5) $\vec{r}\,'(t) = \langle 2t, 3, \frac{1}{t}\rangle$

$\vec{r}\,'(2) = \langle 4, 3, \frac{1}{2}\rangle$

6) $\int_1^2 \vec{r}(t)\,dt = \int_1^2 \langle t^2+2t,\ t^3+3,\ e^{2t}\rangle\,dt$

$= \left\langle \int_1^2 t^2+2t\,dt,\ \int_1^2 t^3+3\,dt,\ \int_1^2 e^{2t}\,dt \right\rangle$

$= \left\langle \frac{t^3}{3}+t^2\Big|_1^2,\ \frac{t^4}{4}+3t\Big|_1^2,\ \frac{1}{2}e^{2t}\Big|_1^2 \right\rangle$

$= \left\langle \left(\frac{8}{3}+4\right)-\left(\frac{1}{3}+1\right),\ \left(\frac{16}{4}+6\right)-\left(\frac{1}{4}+3\right),\ \frac{e^4}{2}-\frac{e^2}{2} \right\rangle$

$= \left\langle \frac{7}{3}+3,\ 7-\frac{1}{4},\ \frac{e^4-e^2}{2} \right\rangle$

7) $$\vec{v}(t) = \int \vec{a}(t)\,dt$$

$$= \int \langle 0,2,5\rangle\,dt$$

$$= \langle 0, 2t, 5t\rangle + \langle c_1, c_2, c_3\rangle$$

$$\underbrace{\langle 1,1,-1\rangle}_{\text{given}} = \vec{v}(0) = \underbrace{\langle 0,0,0\rangle + \langle c_1, c_2, c_3\rangle}_{\text{calculated}}$$

$$\Rightarrow \quad \langle 1,1,-1\rangle = \langle c_1, c_2, c_3\rangle$$

Thus our velocity function is

$$\vec{V}(t) = \langle 1, 2t+1, 5t-1\rangle$$

Now calulate

$$\vec{r}(t) = \int \vec{v}(t)\,dt$$

$$= \int \langle 1, 2t+1, 5t-1\rangle\,dt$$

$$= \langle t, t^2+t, \tfrac{5}{2}t^2 - t\rangle + \langle c_4, c_5, c_6\rangle$$

$$\underbrace{\langle 1,2,3\rangle}_{\text{given}} = \vec{r}(0) = \underbrace{\langle 0,0,0\rangle + \langle c_4, c_5, c_6\rangle}_{\text{calculated}}$$

$$\Rightarrow \quad \langle c_4, c_5, c_6\rangle = \langle 1,2,3\rangle$$

Thus our position function is

$$\vec{r}(t) = \langle t+1, t^2+t+2, \tfrac{5}{2}t^2 - t + 3\rangle.$$