Homework 5 – MATH 1586 Spring 2018

Recall that to add vectors,

$$\langle a, b, c\rangle + \langle d, e, f\rangle = \langle a + d, b + e, c + f\rangle.$$

For a real number α, scalar multiplication obeys

$$\alpha\langle a, b, c\rangle = \langle \alpha a, \alpha b, \alpha c\rangle.$$

The dot product is given by

$$\langle a, b, c\rangle \cdot \langle d, e, f\rangle = ad + be + cf.$$

Recall the determinant of a 2×2 (i.e. two rows, two columns) is

$$\det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc.$$

Recall the determinant of a 3×3 matrix can be computers

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \begin{bmatrix} d & e \\ g & h \end{bmatrix}.$$

Also recall the "basis vectors" $\vec{i} = \langle 1, 0, 0\rangle$, $\vec{j} = \langle 0, 1, 0\rangle$, and $\vec{k} = \langle 0, 0, 1\rangle$. The cross product of two vectors is given by

$$\langle a, b, c\rangle \times \langle d, e, f\rangle = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ a & b & c \\ d & e & f \end{bmatrix}.$$

Recall that differentiation and integration work componentwise: if $\vec{r}(t) = \langle f(t), g(t), h(t)\rangle$, then

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{r}(t) = \langle f'(t), g'(t), h'(t)\rangle,$$

and

$$\int_a^b \vec{r}(t)\mathrm{d}t = \left\langle \int_a^b f(t)\mathrm{d}t, \int_a^b g(t)\mathrm{d}t, \int_a^b h(t)\mathrm{d}t \right\rangle.$$

1. Calculate $\langle 3, 7, 10\rangle + 2\langle 3, 1, -1\rangle$.
2. Compute the dot product $\langle 1, 6, 2\rangle \cdot \langle 7, 1, 3\rangle$.
3. Compute the determinant $\det \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$.
4. Compute the cross product $\langle 1, 0, 1\rangle \times \langle 2, 1, 3\rangle$.
5. Let $\vec{r}(t) = \langle t^2 + 1, 3t - 2, \ln(t)\rangle$. Compute $\vec{r}\,'(2)$.
6. Let $\vec{r}(t) = \langle t^2 + 2t, t^3 + 3, e^{2t}\rangle$. Compute $\displaystyle\int_1^2 \vec{r}(t)\mathrm{d}t$.
7. Suppose that a particle has an initial position of $\vec{r}(0) = \langle 1, 2, 3\rangle$, initial velocity $\vec{v}(0) = \langle 1, 1, -1\rangle$, and obeys acceleration $\vec{a}(t) = \langle 0, 2, 5\rangle$. Find the position of the particle at time t.