Homework 3 — MATH 1586 Spring 2018

Recall the technique of integration by parts is of the form

$$\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$$

Also recall that we derived the antiderivative of the natural logarithm using integration by parts:

$$\int \ln(x)\mathrm{d}x = x\ln(x) - x + C.$$

Recall that an "improper integral" of the form $\int_a^\infty$ or of the form $\int_{-\infty}^b$ is understood in the following way:

$$\int_a^\infty f(x)\mathrm{d}x = \lim_{b\to\infty}\int_a^b f(x)\mathrm{d}x.$$

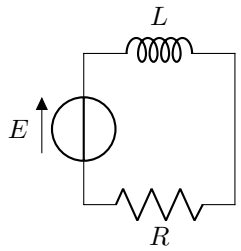
and

$$\int_{-\infty}^b f(x)\mathrm{d}x = \lim_{a\to-\infty}\int_a^b f(x)\mathrm{d}x.$$

Finally, recall the definition of the Laplace transform of a function f:

$$\mathscr{L}\{f\}(x) = \int_0^\infty f(t)e^{-xt}\mathrm{d}t.$$

1. ("RL-Circuit problem")



In a series circuit containing only a resistor (R, measured in ohms), an inductor (L, measured in henries which are ohm $\cdot$ sec), and a current at time t, $i(t)$ (measured in amps), Kirchoff's second law states that the sum of the voltage drop across the inductor $\left(L\dfrac{\mathrm{d}i}{\mathrm{d}t}\right)$ and the voltage drop across the resistor (iR) is the same as the impressed voltage ($E(t)$) on the circuit. We obtain the differential equation for the current $i(t)$

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + Ri(t) = E(t),$$

where L and R are constants known as the inductance and resistance.

A 25-volt electromotive force is applied to a series circuit in which the inductance is 1 henry and the resistance is 30 ohms. $i' + 30i = 25$

a.) What is the differential equation we must solve here?

b.) Calculate $\frac{\mathrm{d}}{\mathrm{d}t}\left[e^{30t}i(t)\right]$. What do you notice about this compared to the left-hand side of your differential equation?

c.) Multiply your differential equation on both sides by e^{30t} and then rewrite the left-hand side as $\frac{\mathrm{d}}{\mathrm{d}t}\left[e^{30t}i(t)\right]$.

d.) Solve the differential equation by integrating and solving for $i(t)$.

2. Compute $\int_5^{11} xe^x \mathrm{d}x$.

3. Compute $\int x^2 e^x \mathrm{d}x$.

4. Compute $\int (2x+3)e^{2x} \mathrm{d}x$.

5. Compute $\int_2^3 \ln(x) \mathrm{d}x$.

6. Let $f(t) = e^{7t}$.

   (a) What integral must you solve in order to calculate $\mathscr{L}\{f\}(x)$?

   (b) Calculate $\mathscr{L}\{f\}(x)$ as an improper integral.

7. Let $f(t) = t$.

   (a) Let $x > 0$. What integral must you solve in order to calculate $\mathscr{L}\{f\}(x)$?

   (b) Calculate $\mathscr{L}\{f\}(x)$ using integration by parts and improper integration.