1. Compute $\int e^{12x}\mathrm{d}x$.

   Solution: Let $u = 12x$, then $\mathrm{d}u = 12\mathrm{d}x$ and so $\frac{1}{12}\mathrm{d}u = \mathrm{d}x$. Therefore, compute

$$\begin{aligned} \int e^{12x}\mathrm{d}x &= \frac{1}{12}\int e^u \mathrm{d}u \\ &= \frac{1}{12}e^u + C \\ &= \frac{1}{12}e^{12x} + C. \end{aligned}$$

2. Compute $\int (t^3 - 2t)^{25}(3t^2 - 2)\mathrm{d}t$.

   Solution: Let $u = t^3 - 2t$, then $\mathrm{d}u = 3t^2 - 2\mathrm{d}t$. Therefore, compute

$$\begin{aligned} \int (t^3 - 2t)^{25}(3t^2 - 2)\mathrm{d}t &= \int u^{25}\mathrm{d}u \\ &= \frac{u^{26}}{26} + C \\ &= \frac{(t^3 - 2t)^{26}}{26} + C. \end{aligned}$$

3. Compute $\int \frac{z^4}{1 - z^5}\mathrm{d}z$.

   Solution: Let $u = 1 - z^5$, then $\mathrm{d}u = -5z^4\mathrm{d}z$, and so $-\frac{1}{5}\mathrm{d}u = z^4\mathrm{d}z$. Therefore, compute

$$\begin{aligned} \int \frac{z^4}{1 - z^5}\mathrm{d}z &= -\frac{1}{5}\int \frac{1}{u}\mathrm{d}u \\ &= -\frac{1}{5}\ln(u) + C \\ &= -\frac{1}{5}\ln(1 - z^5) + C. \end{aligned}$$

4. Compute $\int \frac{5}{w - 3}\mathrm{d}w$.

   Solution: Let $u = w - 3$, then $\mathrm{d}u = \mathrm{d}w$. Therefore, compute

$$\begin{aligned} \int \frac{5}{w - 3}\mathrm{d}w &= 5\int \frac{1}{u}\mathrm{d}u \\ &= 5\ln(u) + C \\ &= 5\ln(w - 3) + C. \end{aligned}$$

5. Compute $\int q^2 e^{q^3-1} \mathrm{d}q$.

Solution: Let $u = q^3 - 1$, then $\mathrm{d}u = 3q^2\mathrm{d}q$ and so $\frac{1}{3}\mathrm{d}u = q^2\mathrm{d}q$. Therefore, compute

$$\begin{aligned}\int q^2 e^{q^3-1}\mathrm{d}q &= \frac{1}{3}\int e^u \mathrm{d}u \\ &= \frac{1}{3}e^u + C \\ &= \frac{1}{3}e^{q^3-1} + C.\end{aligned}$$

6. In calm waters, the oil spilling from the ruptured hull of a grounded tanker forms an oil slick that is circular in shape. If the radius r of the circle is increasing at a rate of

$$r'(t) = \frac{30}{\sqrt{2t+4}} \ \frac{\text{ft}}{\text{min}}$$

t minutes after the rupture occurs, find an expression for the radius at any time t. How large is the polluted area 16 minutes after the rupture occurred? (*note:* $r(0) = 0$)

Solution: We are told $r'(t)$, so to find $r(t)$, we must integrate (with respect to t). Let $u = 2t + 4$, then $\mathrm{d}u = 2\mathrm{d}t$ and so $\frac{1}{2}\mathrm{d}u = \mathrm{d}t$. Therefore, compute

$$\begin{aligned}r(t) &= \int \frac{30}{\sqrt{2t+4}}\mathrm{d}t \\ &= \frac{30}{2}\int \frac{1}{\sqrt{u}}\mathrm{d}u \\ &= 15\int u^{-\frac{1}{2}}\mathrm{d}u \\ &= 15\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C \\ &= 30\sqrt{2t+4} + C.\end{aligned}$$

<u>Find C</u>

Using the given information that $r(0) = 0$, we see

$$\underbrace{0}_{\text{given}} = r(0) = \underbrace{30\sqrt{2(0)+4} + C}_{\text{calculated}}.$$

This yields $0 = 30\sqrt{4} + C = 60 + C$. Therefore solve for C to get $C = -60$. Thus our model is

$$r(t) = 30\sqrt{2t+4} - 60.$$

<u>16 minutes after rupture</u>

Here just calculate $r(16)$:

$$r(16) = 30\sqrt{2(16)+4} - 60 = 30\sqrt{36} - 60 = 180 - 60 = 120\,\text{ft}.$$

7. Suppose that in a certain country, the life expectancy at birth of a female is changing at the rate of

$$g'(t) = \frac{5.45218}{(1+1.09t)^{0.9}}\ \frac{\text{years}}{\text{year}}.$$

Here t is measured in years with $t = 0$ corresponding to the beginning of the year 1900. Find an expression $g(t)$ giving the life expectancy at birth (in years) of a female in that country if the life expectancy at the beginning of 1900 is 50.02 years. What is the life expectancy at birth of a female born in the year 2000 according to this model?

Solution: We are told $g'(t)$, so we must integrate (with respect to t) to find $g(t)$. Let $u = 1 + 1.09t$ so $\mathrm{d}u = 1.09\mathrm{d}t$ and thus $\frac{1}{1.09}\mathrm{d}u = \mathrm{d}t$. Therefore, calculate

$$\begin{aligned} g(t) &= \int \frac{5.45218}{(1+1.09t)^{0.9}}\mathrm{d}t \\ &= \frac{5.45218}{1.09}\int \frac{1}{u^{0.9}}\mathrm{d}u \\ &= 5.002\int u^{-0.9}\mathrm{d}u \\ &= 5.002\frac{u^{0.1}}{0.1} + C \\ &= 50.02(1+1.09t)^{0.1} + C. \end{aligned}$$

Find C

Since $t = 0$ corresponds to the year 1900 and we are told that life expectancy in 1900 was 50.02 years, we see that we were told that $g(0) = 50.02$. Therefore,

$$\underbrace{50.02}_{\text{given}} = g(0) = \underbrace{50.02(1+1.09(0))^{0.1} + C}_{\text{calculated}}.$$

This equation yields $50.02 = 50.02 + C$, or in other words, $C = 0$. Therefore our model is

$$g(t) = 50.02(1+1.09t)^{0.1}.$$

Life expectancy in year 2000

Since $t = 0$ corresponds to 1900, then $t = 100$ corresponds to 2000. Plug this into the model to get

$$g(100) = 50.02(1+1.09(100))^{0.1} = 80.0356\,\text{years}.$$