Homework 1 — MATH 1586 Spring 2018

Recall that for $n \neq -1$,

$$\int x^n \mathrm{d}x = \frac{x^{n+1}}{n+1} + C.$$

Recall that

$$\int \frac{1}{x} \mathrm{d}x = \ln(x) + C.$$

Recall that

$$\int e^x \mathrm{d}x = e^x + C.$$

Recall that

$$\int f(x) + g(x) \mathrm{d}x = \int f(x) \mathrm{d}x + \int g(x) \mathrm{d}x.$$

Finally recall that if F represents any antiderivative of f, then

$$\int_a^b f(x) \mathrm{d}x = F(b) - F(a).$$

Compute the following definite and indefinite integrals.

1. $\int x^{17} \mathrm{d}x$

   Solution: Calculate

$$\int x^{17} \mathrm{d}x = \frac{x^{18}}{18} + C.$$

2. $\int x^3 + 3x^2 + \frac{x}{9} + 1 \mathrm{d}x$

   Solution: Calculate

$$\int x^3 + 3x^2 + \frac{x}{9} + 1 \mathrm{d}x = \frac{x^4}{4} + x^3 + \frac{x^2}{18} + x + C.$$

3. $\int \frac{e^x}{10} + x^2 + \frac{9}{x} + 1 \mathrm{d}x$

   Solution: Calculate

$$\begin{aligned} \int \frac{e^x}{10} + x^2 + \frac{9}{x} + 1 \mathrm{d}x \quad & + \frac{1}{10} \int e^x \mathrm{d}x + \int x^2 \mathrm{d}x + 9 \int \frac{1}{x} \mathrm{d}x + \int 1 \mathrm{d}x \\ & = \frac{e^x}{10} + \frac{x^3}{3} + 9\ln(x) + x + C. \end{aligned}$$

4. $\int_0^1 4x^4 + 3x^2 + 1 \mathrm{d}x$

   Solution: Calculate

$$\begin{aligned} \int_0^1 4x^4 + 3x^2 + 1 \mathrm{d}x \quad & = \left. \frac{4x^5}{5} + x^3 + x \right|_0^1 \\ & = \left(\frac{4}{5} + 1^3 + 1 \right) - \left(\frac{4(0^5)}{5} + 0^3 + 0 \right) \\ & = \frac{14}{5}. \end{aligned}$$

5. $\displaystyle\int_{-3}^{4} -3e^x + \frac{1}{x} + x^5 - 2x^2 \mathrm{d}x$
Solution: Calculate

$$\begin{aligned}
&\int_{-3}^{4} -3e^x + \frac{1}{x} + x^5 - 2x^2 \mathrm{d}x \\
&\quad = -3e^x + \ln(x) + \frac{x^6}{6} - \frac{2x^3}{3}\Bigg|_{-3}^{4} \\
&\quad = \left(-3e^4 + \ln(4) + \frac{4^6}{6} - \frac{2(4^3)}{3}\right) - \left(-3e^{-3} + \ln(-3) + \frac{(-3)^6}{6} - \frac{2(-3)^3}{3}\right).
\end{aligned}$$

(*note: "*$\ln(-3)$*" is a nebulous concept – technically we should not be able to integrate this as I did above because of the singularity at* $x = 0$ *– oops! If the lower and upper bounds were positive, the technique would work fine.*)

6. Is it accurate to say that

$$(*) \qquad \int e^{7x} \mathrm{d}x = e^{7x} + C?$$

How can you use differentiation to check if this is an accurate statement?
Solution: No! It is easy to see that $e^{7x} + C$ is not the antiderivative of e^{7x}. It can be checked by differentiating $e^{7x} + C$:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[e^{7x} + C\right] = 7e^{7x}.$$

This shows that the formula $(*)$ is not correct, because the right-hand side would have to equal e^{7x} (i.e. it would have to be the function inside the integral).