HW 11 (MATH 1586)

① $\frac{\partial \vec{r}}{\partial u} = \langle 1, 0, v \rangle \qquad \frac{\partial \vec{r}}{\partial \vec{v}} = \langle 0, 1, u \rangle$

$$\frac{\partial \vec{r}}{\partial \vec{u}} \times \frac{\partial \vec{r}}{\partial \vec{v}} = \langle 1, 0, v \rangle \times \langle 0, 1, u \rangle$$

$$= \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & v \\ 0 & 1 & u \end{bmatrix}$$

$$= \vec{i} \det \begin{bmatrix} 0 & v \\ 1 & u \end{bmatrix} - \vec{j} \det \begin{bmatrix} 1 & v \\ 0 & u \end{bmatrix} + \vec{k} \det \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \vec{i}(0-v) - \vec{j}(u-0) + \vec{k}(1-0)$$

$$= \langle -v, -u, 1 \rangle$$

So,

$$\left\| \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} \right\| = \sqrt{(-v)^2 + (-u)^2 + 1^2} = \sqrt{v^2 + u^2 + 1}$$

Therefore,

$$\iint_S x \, dS = \int_0^1 \int_0^1 u \sqrt{v^2 + u^2 + 1} \, du \, dv$$

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② $\frac{\partial \vec{r}}{\partial u} = \langle v, 2u, 0 \rangle \quad \frac{\partial \vec{r}}{\partial v} = \langle u, 0, 2v \rangle \Rightarrow \frac{\partial \vec{r}}{\partial \vec{u}} \times \frac{\partial \vec{r}}{\partial \vec{v}} = \cdots = \langle 4uv, -2v^2, -2u^2 \rangle$

Thus,

$$\iint_S \langle y, x, xyz \rangle d\vec{S} = \int_0^1 \int_0^1 \langle u^2, uv, v^2 \rangle \cdot \langle 4uv, -2v^2, -2u^2 \rangle \, du \, dv$$

$$= \int_0^1 \int_0^1 4u^3 v - 2uv^3 - 2u^2 v^2 \, du \, dv$$