Homework 11 — MATH 1586 Spring 2018

Recall that if S is a surface parametrized by $\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v)\rangle$ for $a \le u \le b$ and $c \le v \le d$, then we have two kinds of surface integral: first the scalar surface integral

$$\iint_S f(x,y,z)\mathrm{d}S = \int_a^b \int_c^d f(x(u,v),y(u,v),z(u,v)) \left\| \frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} \right\| \mathrm{d}v\mathrm{d}u,$$

and a vector field surface integral

$$\iint_S \vec{F} \cdot \mathrm{d}\vec{S} = \int_a^b \int_c^d \vec{F}(x(u,v),y(u,v),z(u,v)) \cdot \left(\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v} \right) \mathrm{d}v\mathrm{d}u.$$

1. Let $f(x,y,z) = x$ and let $\vec{r}(u,v) = \langle u, v, uv\rangle$ for $0 \le u \le 1$ and $0 \le v \le 1$. Set up **but do not evaluate** $\displaystyle\iint_S f(x,y,z)\mathrm{d}S$.

2. Let $\vec{F} = \langle y, x, xyz\rangle$ and let $\vec{r}(u,v) = \langle uv, u^2, v^2\rangle$ for $0 \le u \le 1$ and $0 \le v \le 1$. Set up **but do not evaluate** $\displaystyle\iint_S \vec{F} \cdot \mathrm{d}\vec{S}$.