HW 10 MATH 1586

① $\vec{r}\,'(t) = \langle 2t, 1 \rangle \Rightarrow \|\vec{r}\,'(t)\| = \sqrt{4t^2+1}$

$0 \le t \le 2$

Therefore,

$$\int_C x\,ds = \int_0^2 2t\sqrt{4t^2+1}\,dt$$

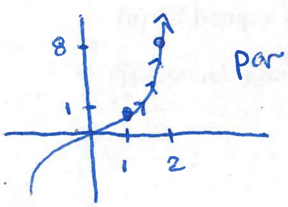
$u = 4t^2+1$ $\quad \boxed{t=2 \to u=17}$

$du = 8t\,dt$ $\quad \boxed{t=0 \to u=1}$

$\frac{1}{8}du = dt$

$$= \frac{2}{8}\int_1^{17} u^{1/2}\,du = \frac{1}{4}\,\frac{u^{3/2}}{3/2}\Big|_1^{17} = \frac{2}{12}\left(17^{3/2}-1\right)$$

$$= \frac{1}{6}\left(17^{3/2}-1\right)$$

② 


parametrize as

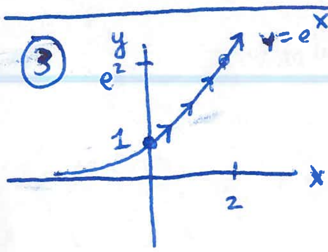
$\vec{r}(t) = \langle t, t^3 \rangle$

$1 \le t \le 2$

So,

$$\int_C xy + \ln(x)\,dy = \int_1^2 \left(t\cdot t^3 + \ln(t)\right)\overbrace{3t^2}^{y'(t)}\,dt$$

$$= 3\int_1^2 \left(t^4 + \ln(t)\right)t^2\,dt$$

③ 


Parametrize as

$\vec{r}(t) = \langle t, e^t \rangle$

$0 \le t \le 2$

So,

$$\int_C x e^{xy}\,dx = \int_0^2 \left(t e^{te^t}\right)\overbrace{(1)}^{x'(t)}\,dt$$

④ $\vec{r}\,'(t) = \langle 2t, -3t^2 \rangle$

$0 \le t \le 1$

So,

$$\int_C \vec{F}\cdot d\vec{r} = \int_0^1 \langle \underbrace{(t^2)^2}_{t^4}\underbrace{(-t^3)^3}_{-t^6}, -(-t^3)\sqrt{t^2} \rangle \cdot \overbrace{\langle 2t, -3t^2 \rangle}^{\vec{r}\,'(t)}\,dt$$

$$= \int_0^1 \langle -t^{10}, t^4 \rangle \cdot \langle 2t, -3t^2 \rangle\,dt$$

$$= \int_0^1 -2t^{11} - 3t^6\,dt = -\frac{2}{12}t^{12} - \frac{3}{7}t^7\Big|_0^1 = -\frac{1}{6} - \frac{3}{7}$$

5 $\vec{r}\,'(t) = \langle 3t^2, -2t, 1 \rangle$

$0 \le t \le 1$

So,

$$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 \langle \sin(t^3), \cos(-t^2), t^4 \rangle \cdot \langle 3t^2, -2t, 1 \rangle \, dt$$

$$= \int_0^1 3t^2 \sin(t^3) - 2t\cos(-t^2) + t^4 \, dt$$