Homework 10 — MATH 1586 Spring 2018

Let $C$ be a curve that is parameterized by $\vec{r}(t)$ for $a \leq t \leq b$. Recall that the $\mathrm{d}s$-type line integral is:

$$\int_C f(x,y,z)\mathrm{d}s = \int_a^b f(x(t),y(t))\|\vec{r}\,'(t)\|\mathrm{d}t,$$

the $\mathrm{d}x$-type is

$$\int_C f(x,y,z)\mathrm{d}x = \int_a^b f(x(t),y(t))x'(t)\mathrm{d}t,$$

the $\mathrm{d}y$-type is

$$\int_C f(x,y,z)\mathrm{d}y = \int_a^b f(x(t),y(t))y'(t)\mathrm{d}t,$$

and the vector field line integral is

$$\int_C \vec{F}\cdot\mathrm{d}\vec{r} = \int_a^b \vec{F}(\vec{r}(t))\cdot\vec{r}\,'(t)\mathrm{d}t.$$

Also recall the "tricks" to parametrize the line segment between two points: the line segment from the point $(a,b)$ to the point $(c,d)$ can be parametrized by $\vec{r}(t) = (1-t)\langle a,b\rangle + t\langle c,d\rangle$ for $0 \leq t \leq 1$. To parametrize a curve given by a function $y = h(t)$, use $\vec{r}(t) = \langle t, h(t)\rangle$ and choose $a \leq t \leq b$ appropriately.

1. Compute $\displaystyle\int_C x\mathrm{d}s$ where $C$ is the curve given by $\vec{r}(t) = \langle t^2, t\rangle 0 \leq t \leq 2$.

2. Set up **but do not evaluate** the following line integral: $\displaystyle\int_C (xy + \ln(x))\mathrm{d}y$ where $C$ is the arc of the function $y = x^3$ from $(1,1)$ to $(2,8)$.

3. Set up **but do not evaluate** the following line integral: $\displaystyle\int_C xe^{xy}\mathrm{d}x$, where $C$ is the arc of the curve $y = e^x$ from $(0,1)$ to $(2,e^2)$.

4. Calculate $\displaystyle\int_C \vec{F}\mathrm{d}\vec{r}$, where $\vec{F} = \langle x^2y^3, -y\sqrt{x}\rangle$ and $\vec{r}(t) = \langle t^2, -t^3\rangle$ and $0 \leq t \leq 1$.

5. Set up **but do not evaluate** the following line integral: $\displaystyle\int_C \vec{F}\mathrm{d}\vec{r}$ where $\vec{F} = \langle \sin(x), \cos(y), xz\rangle$ and $C$ is the curve $\vec{r}(t) = \langle t^3, -t^2, t\rangle$ and $0 \leq t \leq 1$.