§7.4 #11

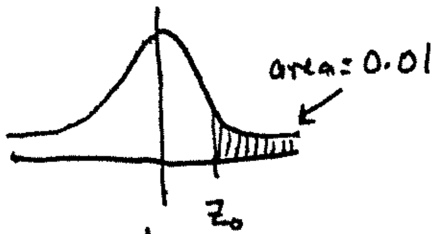
claim → $H_0: p \le 0.75$

$H_a: p > 0.75$

right-tailed

$\alpha = 0.01$
$n = 150$
$\hat{p} = 0.77$

$q = 1 - p = 1 - 0.75 = 0.25$



$z_0 = 2.33$

↓

rejection region

$z > 2.33$

test statistic

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}}$$

$$\sigma_{\hat{p}} = \sqrt{\frac{(0.75)(0.25)}{150}}$$

$$= 0.0353$$

$$\boxed{z = \frac{0.77 - 0.75}{0.0353} = 0.566}$$

↓

fail to reject Ho

↓

"There is not sufficient evidence to reject the claim."

§7.4 #16

claim → $\begin{cases} H_0 : p = 0.30 \\ H_a : p \neq 0.30 \end{cases}$

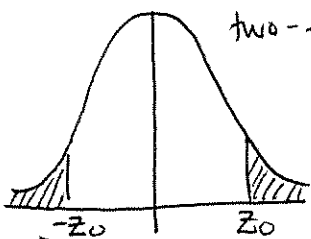
$\alpha = 0.05$

$n = 200$

$\hat{p} = \frac{72}{200} = 0.36$

$q = 1 - p = 1 - 0.30 = 0.70$

two-tailed



area = 0.05

area of each = 0.025

table → $-z_0 = -1.96$

$z_0 = 1.96$

$\boxed{\text{Rejection Region: } z > 1.96 \text{ or } z < -1.96}$

Test Statistic:

$$z = \frac{\hat{p} - p}{\sigma_{\hat{p}_n}}$$

$$\sigma_{\hat{p}_n} = \sqrt{\frac{pq}{n}}$$

$$\sigma_{\hat{p}_n} = \sqrt{\frac{0.3(0.7)}{200}} = 0.0324$$

$$\boxed{z = \frac{0.36 - 0.3}{0.0324} = 1.8516}$$

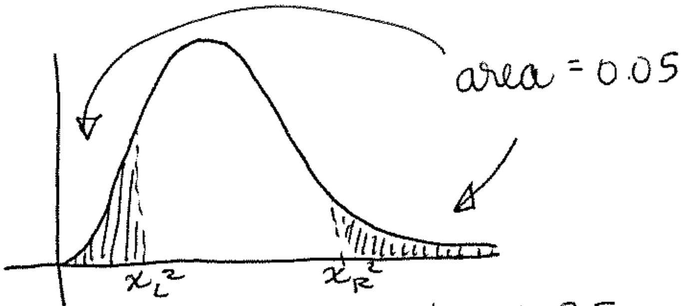
→ Fail to reject H_0

→ "There isn't sufficient evidence to reject the claim."

§7.5 #18

claim → $\begin{cases} H_0 : \sigma^2 \overset{?}{=} 1.0 \\ H_a : \sigma^2 \neq 1.0 \end{cases}$

$\left[\begin{array}{l} \alpha = 0.05 \\ n = 25 \\ s^2 = 1.65 \end{array} \right] \rightarrow df = 24$



area of each = 0.025

table → $\chi_L^2 = 12.401$

$\chi_R^2 = 39.364$

→ $\boxed{\text{Rejection Region: } \chi^2 < 12.401 \text{ or } \chi^2 > 39.364}$

Test - Statistic:

$$\left[\chi^2 = \frac{(n-1)s^2}{\sigma^2} \right] \rightarrow \chi^2 = \frac{(25-1)(1.65)}{1.0} = \boxed{39.6}$$

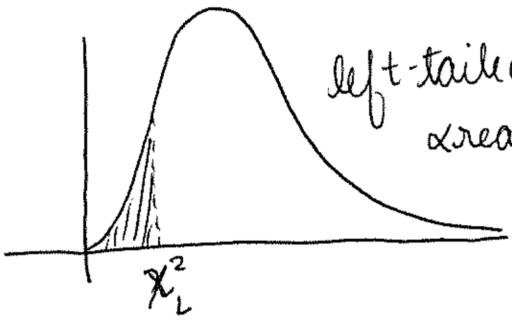
→ Reject H_0

→ "There is not sufficient evidence to support the claim."

§7.5 #19

$$\begin{cases} H_0: \sigma \geq 36 \\ H_a: \sigma < 36 \end{cases} \leftarrow \text{claim}$$

$$\begin{bmatrix} \alpha = 0.1 \\ n = 22 \\ s = 33.4 \end{bmatrix} \rightarrow df = 21$$



left-tailed

area = 0.1

table $\rightarrow \chi^2_L = 13.240$

$\rightarrow$ $\boxed{\text{Rejection Region: } \chi^2 < 13.240}$

Test-Statistic

$$\left[\chi^2 = \frac{(n-1)s^2}{\sigma^2} \right] \rightarrow \chi^2 = \frac{(22-1)(33.4)^2}{36^2} = \boxed{18.0762}$$

$\rightarrow$ Fail to reject H_0

$\rightarrow$ "There is not sufficient evidence to support the claim."

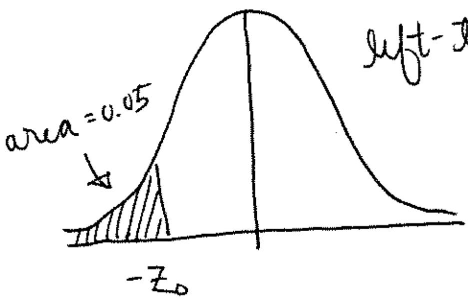
§8.1 #17

$\mu_1 \to$ Region A, $\mu_2 \to$ Region B

$$\begin{cases} H_0: \mu_1 \geq \mu_2 \\ H_a: \mu_1 < \mu_2 \end{cases}$$

claim $\to$ H_a

$\alpha = 0.05$
$\bar{x}_1 = 14.0$
$\bar{x}_2 = 15.1$
$\sigma_1 = 2.9$
$\sigma_2 = 3.3$
$n_1 = 60$
$n_2 = 60$



left-tailed

table $\to$ $z_0 = -1.645$

Rejection Region:
$z < -1.645$

Test-statistic

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}}$$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{(2.9)^2}{60} + \frac{(3.3)^2}{60}} = 0.5672$$

$$\to z = \frac{(14.0 - 15.1) - 0}{0.5672} = \boxed{-1.9395}$$

$\to$ Reject H_0

$\to$ "There is sufficient evidence to support the claim."

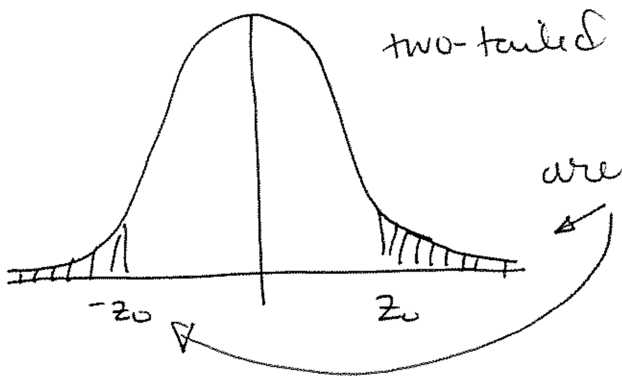
§8.1 #21

$\mu_1 \rightarrow$ Spring, TX $\quad \mu_2 \rightarrow$ Austin, TX

$\alpha = 0.01$
$\bar{x}_1 = 127{,}414$
$\bar{x}_2 = 112{,}301$
$\sigma_1 = 25{,}875$
$\sigma_2 = 27{,}110$
$n_1 = 25$
$n_2 = 25$

claim $\rightarrow \begin{cases} H_0 : \mu_1 = \mu_2 \\ H_a : \mu_1 \neq \mu_2 \end{cases}$

two-tailed



total area = 0.01
area of each = 0.005
$\rightarrow$ table $\rightarrow -z_0 = -2.575$
$z_0 = 2.575$

Rejection Region:
$z > 2.575$ or $z < -2.575$

Test-Statistic

$$\left[\begin{array}{l} z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1 - \bar{x}_2}} \\ \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}} \end{array}\right] \rightarrow$$

$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{(25{,}875)^2}{25} + \frac{(27{,}110)^2}{25}} = 7495.24E$$

$$z = \frac{(127{,}414 - 112{,}301) - 0}{7495.2458} = \boxed{2.0163}$$

$\rightarrow$ Fail to reject H_0

$\rightarrow$ "There is not sufficient evidence to reject the claim."

§8.2 #14

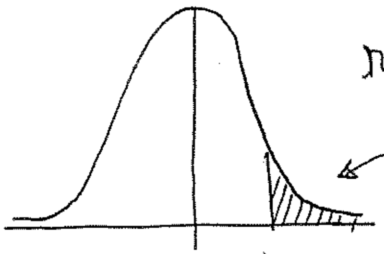
$\mu_1 \to$ Burger Shop , $\mu_2 \to$ Fry World

$\alpha = 0.05$
$\bar{x}_1 = 5.46$
$s_1 = 0.89$
$n_1 = 22$
$\bar{x}_2 = 5.12$
$s_2 = 0.79$
$n_2 = 30$

pop. variances are equal $\to df = 22+30-2 = 50$

$$\begin{cases} H_0 : \mu_1 \le \mu_2 \\ H_a : \mu_1 > \mu_2 \end{cases}$$

claim $\to H_a$



right-tailed

area = 0.05

table $\to t_0 = 1.676$

Rejection Region: $t > 1.676$

Test-statistic

Since pop. variances are equal

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}}$$

$$s_{\bar{x}_1 - \bar{x}_2} = \hat{\sigma}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\hat{\sigma} = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$$

$$\hat{\sigma} = \sqrt{\frac{(22-1)(0.89)^2 + (30-1)(0.79)^2}{22+30-2}}$$

$= 0.8335$

$$s_{\bar{x}_1 - \bar{x}_2} = (0.8335)\sqrt{\frac{1}{22} + \frac{1}{30}}$$

$= 0.2339$

$$t = \frac{(5.46 - 5.12) - 0}{0.2339}$$

$\boxed{t = 1.4533}$

$\to$ fail to reject H_0

$\to$ "There is not sufficient evidence to support the claim."

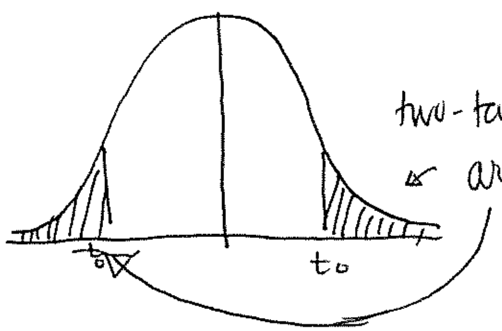
§8.2 #18

$\mu_1 \to$ Kauai Co. , $\mu_2 \to$ Maui Co.

claim $\to$ $\begin{cases} H_0 : \mu_1 = \mu_2 \\ H_a : \mu_1 \neq \mu_2 \end{cases}$

$\alpha = 0.10$
$\bar{x}_1 = 56{,}900$
$s_1 = 12{,}100$
$n_1 = 18$
$\bar{x}_2 = 57{,}800$
$s_2 = 8000$
$n_2 = 20$

because pop. variances are <u>not</u> equal:
$\to df = 18 - 1 = 17$



two-tailed
area = 0.1

total area = 0.1
$\to$ area of each = 0.05
table $\to$ $t_0 = 1.740$
$-t_0 = -1.740$

$\to$ **Rejection Region:**
$t > 1.740$ or $t < -1.740$

<u>Test-Statistic</u> Because pop. variances are <u>not</u> equal $\to$

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1 - \bar{x}_2}}$$

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$\downarrow$

$$s_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{(12{,}100)^2}{18} + \frac{(8000)^2}{20}}$$

$= 3366.5842$

$$t = \frac{(56{,}900 - 57{,}800) - 0}{3366.5842}$$

$\boxed{t = -0.2673}$

$\to$ Fail to reject H_0

$\to$ "There is not enough evidence to reject the claim."

§8.3 #11 Given:

$$\begin{cases} H_0: \mu_d \le 0 \\ H_a: \mu_d > 0 \end{cases}$$

claim → $H_a: \mu_d > 0$

$n = 10$

$\left.\begin{matrix} \alpha = 0.01 \\ \text{d.f.} = 9 \end{matrix}\right\}$ → critical value $t_0 = 2.821$

rejection region

$\boxed{t > 2.821}$

Excel: $\begin{cases} \bar{d} = 0.097 \\ s_d \approx 0.043 \end{cases}$

Test statistic: $\frac{s_d}{\sqrt{n}} = \frac{0.043}{\sqrt{10}} = 0.01359$

$$t = \frac{\bar{d} - \mu_d}{s_d/\sqrt{n}} = \frac{0.097 - 0}{0.01359}$$

$$= 7.1376$$

Therefore we reject Ho.

"There is enough evidence to support the claim."

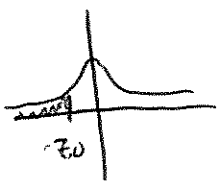
$\boxed{\S 8.4 \ \#12}$

$$\text{claim} \rightarrow \begin{cases} H_0: p_1 \geq p_2 \\ H_a: p_1 < p_2 \end{cases}$$

Midwest	West
$n_1 = 340$	$n_2 = 300$
$\hat{p}_1 = \frac{289}{340} = 0.85$	$\hat{p}_2 = \frac{282}{300} = 0.94$

$\alpha = 0.10$

$\downarrow$



$z_0 = -1.28$

$\downarrow$

$\boxed{\text{Rej rg}: z < -1.28}$

Test statistic

$$\bar{p} = \frac{289 + 282}{340 + 300} = 0.8921$$

$$\bar{q} = 1 - \bar{p} = 0.1079$$

$$\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)} = \sqrt{(0.8921)(0.1079)\left(\frac{1}{340} + \frac{1}{300}\right)}$$

$$= 0.02457$$

$$z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\bar{p}\bar{q}\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{(0.85 - 0.94) - 0}{0.02457}$$

$$= -3.663$$

Therefore reject Ho.

"There is evidence to support the claim."