MATH 1550 - EXAM 2 - FALL 2018
SOLUTION

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Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

Binomial: $P(x) = \dfrac{n!}{(n-x)!x!}p^x q^{n-x}$
Geometric: $P(x) = pq^{x-1}$
Poisson: $P(x) = \dfrac{\mu^x e^{-\mu}}{x!}$
$E = z_c \dfrac{\sigma}{\sqrt{n}}; \bar{x} - E < \mu < \bar{x} + E$
$E = t_c \dfrac{s}{\sqrt{n}}; \bar{x} - E < \mu < \bar{x} + E$
$E = z_c \sqrt{\dfrac{\hat{p}\hat{q}}{n}}; \hat{p} - E < p < \hat{p} + E$
$\dfrac{(n-1)s^2}{\chi_R^2} < \sigma^2 < \dfrac{(n-1)s^2}{\chi_L^2}$
$\sqrt{\dfrac{(n-1)s^2}{\chi_R^2}} < \sigma < \sqrt{\dfrac{(n-1)s^2}{\chi_L^2}}$

Finding χ_R^2 and χ_L^2: use $\dfrac{1+c}{2}$ for χ_L^2 use $\dfrac{1-c}{2}$ for χ_R^2
$\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$
$z = \dfrac{x - \mu}{\sigma}$
$z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}}; \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$
$t = \dfrac{\bar{x} - \mu}{s_{\bar{x}}}; s_{\bar{x}} = \dfrac{s}{\sqrt{n}}$
$z = \dfrac{\hat{p} - p}{\sigma_{\hat{p}}}; \sigma_{\hat{p}} = \sqrt{\dfrac{pq}{n}}$
$\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$

1. (16 points) (a) (8 points) An auto parts seller finds that 1 in every 100 parts sold is defective. Find the probability that the first defective part is the third part sold.

   Solution: This is a geometric distribution with $p = \frac{1}{100}$ and $q = \frac{99}{100}$. The distribution is given by $P(x) = \frac{1}{100}\left(\frac{99}{100}\right)^{x-1}$. Therefore the requested probability is

$$P(3) = \frac{1}{100}\left(\frac{99}{100}\right)^{3-2} = \frac{1}{100}\left(\frac{99}{100}\right)^{1} = 0.0099.$$

   (b) (8 points) The mean number of heart transplants performed per day in the U.S. in a recent year was about six. Find the probability that the number of heart transplants performed on any given day is no more than 1.

   Solution: This is a Poisson distribution with $\mu = 6$. The distribution is given by $P(x) = \frac{6^x e^{-6}}{x!}$. The requested probability is

$$P(x \leq 1) = P(0) + P(1) = \frac{6^0 e^{-6}}{0!} + \frac{6e^{-6}}{1!} = e^{-6} + 6e^{-6} \approx 0.01735$$

2. (15 points) The monthly utility bills in a city are normally distributed, with a mean of \$100 and a standard deviation of \$12. Find the probability that a randomly selected utility bill is between \$90 and \$120.

   Solution: Here $\mu = 100$ and $\sigma = 12$. We want to find $P(90 \leq x \leq 120)$. Subtracting μ and dividing by σ yields

$$\frac{90-100}{12} \leq \underbrace{\frac{x-\mu}{\sigma}}_{=z} \leq \frac{120-100}{12},$$

   i.e.

$$-0.83 \leq z \leq 1.66.$$

   To use the normal table we can compute the desired probability by the following calculation:

$$P(-0.83 \leq z \leq 1.66) = P(z \leq 1.66) - P(-0.83) \approx 0.9515 - 0.2033 = 0.7482.$$

3. (13 points) The time spent (in days) waiting for a kidney transplant for people ages 35-49 can be approximated by a normal distribution with mean 1674 and standard deviation 212.5. What waiting time represented the 80th percentile?

   Solution: Here $\mu = 1674$ and $\sigma = 212.5$. The 80th percentile is P_{80} corresponds to the critical value $z = 0.84$. Therefore we see that

$$x = z\sigma + \mu = (0.84)(212.5) + 1674 = 1852.5 \text{ days}.$$

4. (14 points) During a certain week, the mean price of gasoline in California was \$4.117 per gallon. A random sample of 38 gas stations is selected from this population. What is the probability that the mean price from the sample was between \$4.128 and \$4.143 that week? Assume $\sigma = \$0.049$.

   Solution: Here $\mu_{\bar{x}} = \mu = 4.117$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.049}{\sqrt{38}} = 0.007948$. Now convert the inequality $4.128 \leq \bar{x} \leq 4.143$ into a z score:

$$\frac{4.128-4.117}{0.007948} \leq z \leq \frac{4.143-4.117}{0.007948},$$

   i.e.

$$1.38 \leq z \leq 3.27.$$

   Therefore the requested probability is

$$P(4.128 \leq \bar{x} \leq 4.143) = P(1.38 \leq z \leq 3.27) = P(z \leq 3.27) - P(z \leq 1.38) = 0.0833.$$

5. (14 points) In a random sample of five people, the mean driving distance to work was 22.2 miles and the sample standard deviation was 5.8 miles. Construct a 95% confidence interval for the population mean.

   (a) (8 points) Summarize the data by filling out the table below:

c	$= 0.95$
n	$=5$
d.f.	$= \underline{\mathbf{4}}$
(Critical value) t_c	$= \underline{\mathbf{2.776}}$
$\bar{x}$	$=22.2$
s	$=5.8$
E	$=\underline{\mathbf{t_c \frac{s}{\sqrt{n}} = 2.776 \frac{5.8}{\sqrt{5}} = 7.2004}}$

   (b) (6 points) (Fill in the **three** blanks) The confidence interval is:

$$\underline{\mathbf{\bar{x} - E = 14.9996}} < \underline{\mu} < \underline{\mathbf{\bar{x} + E = 29.4004}}$$

6. (14 points) In a survey of 2303 U.S. adults, 734 believe in UFOs. Construct a 90% confidence interval for the population proportion of U.S. adults who believe in UFOs.

   (a) (8 points) Summarize the data by filling out the table below:

c	$= 0.9$
(Critical value) z_c	$=1.645$
n	$=\underline{\mathbf{2303}}$
$\underline{\mathbf{\hat{p}}}$	$=0.31$
$\hat{q}$	$=\underline{\mathbf{1 - \hat{p} = 1 - 0.31 = 0.69}}$
E	$=\underline{\mathbf{z_c \sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.645 \sqrt{\frac{(0.31)(0.69)}{2303}} = 0.0158}}$

   (b) (6 points) (Fill in the **three** blanks) The confidence interval is:

$$\underline{\mathbf{\hat{p} - E = 0.31 - 0.0158 = 0.2942}} < \underline{\mathbf{p}} < \underline{\mathbf{\hat{p} + E = 0.31 + 0.0158 = 0.3258}}$$

7. (14 points) A magazine includes a report on the prices of subcompact digital cameras. The article states that 11 randomly selected subcompact digital cameras have a sample standard deviation of $109. Use an 80% level of confidence to construct a confidence interval for the population standard deviation.

(a) (8 points) Summarize the data by filling out the table below:

c	$=0.8$
n	$=11$
d.f.	$=\underline{\mathbf{10}}$
$\underline{\mathbf{s}}$	$=109$
$\frac{1+c}{2}$	$=0.9$
$\frac{1-c}{2}$	$= 0.1$
χ^2_R	$=\underline{\mathbf{15.987}}$
χ^2_L	$=\underline{\mathbf{4.865}}$

(b) (6 points) (Fill in the **three** blanks) The confidence interval is:

$$\underline{\sqrt{\frac{\mathbf{(n-1)s^2}}{\chi^2_{\mathbf{R}}}} = \sqrt{\frac{\mathbf{10(109^2)}}{\mathbf{15.987}}} = \mathbf{86.207}} < \underline{\sigma} < \underline{\sqrt{\frac{\mathbf{(n-1)s^2}}{\chi^2_{\mathbf{L}}}} = \sqrt{\frac{\mathbf{10(109^2)}}{\mathbf{4.865}}} = \mathbf{156.237}}$$