MATH 1550 - EXAM 1 FALL 2018
SOLUTION

Friday 14 September 2018
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (6 points) Consider the following data representing the pungencies of tabasco peppers (in thousands of Scoville units):
   35 51 44 42 37 38 36 39
   44 43 40 40 32 39 41 38
   Construct a frequency distribution for the data using 4 classes.
   Solution (this problem resembles multiple problems in **HW1**): First compute

$$\text{range} = \max - \min = 51 - 32 = 19$$

   and

$$\text{class width} = \frac{\text{range}}{\#\text{ of classes}} = \frac{19}{4} = 4.75 \overset{\text{round up}}{\longrightarrow} 5,$$

   we get the following distribution:

Class	Frequency
$32--36$	3
$37--41$	8
$42--46$	4
$47--51$	1

2. (10 points) Consider the following data representing the cholesterol levels of a sample of 9 employees:
   154 240 171 188 235 203 184 181 203
   (a) (5 points) Find the median.
   Solution (this problem resembles **Quiz 1 and #13 in HW2**): First order the data to get

   154 171 181 184 188 203 203 235 240

   From this ordering, the middle data point is 188, so that is the median.
   (b) (5 points) Find the mode.
   Solution (this problem resembles **Quiz 1 and #13 in HW2**): The most common data point is 203, which is the mode.

3. (12 points) Consider the following data representing the total number of votes for the office of President of the United States (in hundreds of thousands) in West Virginia for general elections from 2000 to 2016:
   631.9 750.3 701.3 655.9 678.1
   (a) (6 points) Find the mean.
   Solution (this problem resembles **Quiz 1 and #13 in HW2**): Calculate

$$\text{mean} = \frac{631.9 + 750.3 + 701.3 + 655.9 + 678.1}{5} = \frac{3417.5}{5} = 683.5.$$

   (b) (6 points) Find the population standard deviation.
   Solution (this problem resembles **Quiz 1 and #13 in HW2**): Compute

$$\begin{aligned}\sigma &= \sqrt{\frac{(631.9-683.5)^2 + (750.3-683.5)^2 + (701.3-683.5)^2 + (655.9-683.5)^2 + (678.1-683.5)^2}{5}} \\ &= \sqrt{\frac{8232.56}{5}} \\ &= \sqrt{1646.512} \\ &= 40.577\end{aligned}$$

4. (12 points) (a) (6 points) The mean monthly utility bill for a sample of households in a city is \$68, with a standard deviation of \$7. Between what two values does about 95% of the data lie? (Assume the data has a bell-shaped distribution)
Solution (this problem resembles **#29 in HW2**): We are told that the mean is $\mu = 68$ and the standard deviation is $\sigma = 7$. The empirical rule tells us that 95% of the data lies within two standard deviations of the mean. So compute

$$\mu + 2\sigma = 68 + 2(7) = 68 + 14 = 82$$

and

$$\mu - 2\sigma = 68 - 2(7) = 68 - 14 = 54.$$

Therefore 95% of the data lies between the values \$54 and \$82.

(b) (6 points) You are conducting a survey on the number of people per household in your region. From a sample with $n = 48$, the mean number of pets per household is 2 pets and the standard deviation is 1 pet. Using Chebychev's theorem, determine at least how many of the households have 0 to 4 pets.
Solution (this problem resembles **#36 in HW2**): First figure out how many standard deviations ($s = 1$) away from the mean $\overline{x} = 2$ the values 0 and 4 are. Since

$$2 - 2(1) = 0$$

and

$$2 + 2(1) = 4,$$

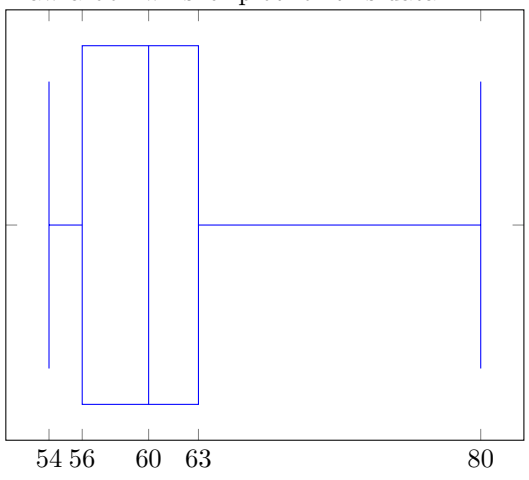
we see that "0 to 4 pets" indicates data values $k = 2$ standard deviations from the mean. Therefore Chebyshev's theorem tells us that $1 - \frac{1}{k^2} = 1 - \frac{1}{4} = \frac{3}{4}$ of the data are in the range of "0 to 4 households". From our sample, we can say that at least $\frac{3}{4}$ of the $n = 48$ houses sampled yields $\left(\frac{3}{4}\right)(48) = 36$ households are in this category.

5. (20 points) Consider the following data points:
56 63 57 60 80 54 63

(a) (6 points) Find the quartiles Q_1, Q_2, and Q_3.
Solution (this problem resembles **#11 and #17 in HW3**):
First order the data to get 54 56 57 60 63 63 80 First find the median to get $Q_2 = 60$. Find the median of the data points to the left of 60 to get $Q_1 = 56$ and find the median of the data points to the right of 60 to get $Q_3 = 63$.

(b) (6 points) Draw a box whisker plot for this data.

Solution:



(c) (4 points) Find the IQR.
Solution:

$$IQR = Q_3 - Q_1 = 63 - 56 = 7$$

(d) (4 points) Identify outliers.
Solution: First compute

$$1.5IQR = 1.5(7) = 10.5,$$

$$Q_1 - 1.5IQR = 56 - 10.5 = 45.5,$$

and

$$Q_3 + 1.5IQR = 63 + 10.5 = 73.5.$$

Since the data point 80 is larger than $Q_3 + 1.5IQR$, we consider it an outlier.

6. (18 points) A probability experiment is carried out by flipping three coins and then rolling a 3-sided die.

(a) (6 points) How many outcomes are in the sample space?
Solution (this problem resembles **#51 in HW3**): 24

(b) (6 points) How many outcomes are in the event of flipping two heads then a tail and then rolling a 3? What is the probability of this event?
Solution: There is one outcome for this event. Therefore the probability is $\frac{1}{24}$.
(*note: there was some confusion on if the coin flips were ordered or not, so I also accepted that there could be* 3 *outcomes for this event which would yield probability* $\frac{3}{24} = \frac{1}{8}$)

(c) (6 points) How many outcomes are in the event of the first coin flip being heads and rolling a number bigger than 1? What is the probability of this event?
Solution: There are 8 outcomes in this event. Therefore the probability of the event is $\frac{8}{24} = \frac{1}{3}$.

7. (11 points) In a survey, 510 adults were asked whether they drive a pickup truck and whether they drive a Ford. The results showed that three in ten adults surveyed drive a Ford. Of the adults surveyed that drive Fords, two in nine drive a pickup truck. Find the probability that a randomly selected adult drives a Ford and drives a pickup truck.
Solution (this problem resembles **Quiz 3 and #20 and # 23 in HW4**): We are told that $P\Big(\text{Ford}\Big) = \frac{3}{10}$ and $P\Big(\text{pickup truck}|\text{Ford}\Big) = \frac{2}{9}$. Therefore using the multiplication rule

$$P\Big(\text{Ford and pickup truck}\Big) = P(\text{Ford})P\Big(\text{pickup truck}|\text{Ford}\Big) = \frac{3}{10} \cdot \frac{2}{9} = \frac{6}{90} \approx 0.0666\ldots.$$

8. (11 points) Of the cartons produced by a company, 5% have a puncture, 8% have a smashed corner, and 0.4% have both a puncture and a smashed corner. Find the probability that a randomly selected carton has a puncture or has a smashed corner.
Solution (this problem resembles **#16 in HW4**): We are told that $P\Big(\text{punctured}\Big) = 0.05$, $P\Big(\text{smashed}\Big) = 0.08$, and that
$P\Big(\text{punctured and smashed}\Big) = 0.004$. Therefore using the addition rule,

$$\begin{aligned} P\Big(\text{punctured or smashed}\Big) &= P\Big(\text{punctured}\Big) + P\Big(\text{smashed}\Big) - P\Big(\text{punctured and smashed}\Big) \\ &= 0.05 + 0.08 - 0.004 \\ &= 0.126. \end{aligned}$$