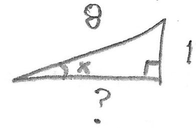
HW8

§9.3 #5 | $\sin(x) = \frac{1}{8}$, x in QI

a) $\sin(2x) = 2\sin(x)\cos(x)$



$\Rightarrow ?^2 + 1^2 = 8^2$

$\Downarrow$

$? = \sqrt{63}$

Thus,

$$\sin(2x) = 2\left(\frac{1}{8}\right)\left(\frac{\sqrt{63}}{8}\right)$$
$$= \frac{\sqrt{63}}{32}$$

b) $$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$= \left(\frac{\sqrt{63}}{8}\right)^2 - \left(\frac{1}{8}\right)^2$$
$$= \frac{62}{64} = \frac{31}{32}$$

c) $$\tan(2x) = \frac{\sin(2x)}{\cos(2x)} = \frac{\sqrt{63}/32}{31/32} = \frac{\sqrt{63}}{31}$$

#11 | $$2\sin\left(\frac{\pi}{4}\right)2\cos\left(\frac{\pi}{4}\right) = 2\left(2\sin\left(\frac{\pi}{4}\right)\cos(\pi/4)\right)$$
$$= 2\left(\sin\left(\frac{\pi}{2}\right)\right)$$
$$= 2$$

#13 | $\sin\left(\frac{\pi}{8}\right)$

Find α so that $\frac{\alpha}{2} = \frac{\pi}{8} \Rightarrow \boxed{\alpha = 2\left(\frac{\pi}{8}\right) = \frac{\pi}{4}}$

Thus,

$$\sin\left(\frac{\pi}{8}\right) = \sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{2}}$$

in QI

$$= + \sqrt{\frac{1-\cos(\pi/4)}{2}}$$
$$= \sqrt{\frac{1-\sqrt{2}/2}{2}}$$

#19| $\tan\left(-\frac{3\pi}{8}\right)$

Soln: Write $-\frac{3\pi}{8}$ as $\frac{\alpha}{2}$:

$$\frac{\alpha}{2} = -\frac{3\pi}{8}$$

$$\Downarrow$$

$$\alpha = 2\left(-\frac{3\pi}{8}\right) = -\frac{3\pi}{4}$$

Therefore,

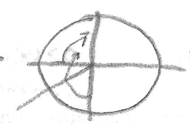
$$\tan\left(-\frac{3\pi}{8}\right) = \frac{\sin\left(-\frac{3\pi}{8}\right)}{\cos(-3\pi/8)} = \frac{\sin\left(\frac{\alpha}{2}\right)}{\cos\left(\frac{\alpha}{2}\right)}$$

in QIII so sin & cos are negative

$$= \frac{-\sqrt{\frac{1-\cos(-3\pi/4)}{2}}}{-\sqrt{\frac{1+\cos(-3\pi/4)}{2}}}$$

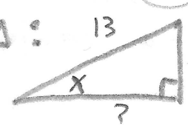
$$= \frac{\sqrt{\frac{1+\sqrt{2}/2}{2}}}{\sqrt{\frac{1-\sqrt{2}/2}{2}}}$$

#21| if $\sin(x) = -\frac{12}{13}$ and x in QIII,



$\Rightarrow \frac{x}{2}$ in QII

first draw a Δ:



$\rightarrow\ ?^2 + 12^2 = 13^2$

$?^2 = 169 - 144 = 25$

$\downarrow$

$? = \sqrt{25} = 5$

Calculate

a) $\sin\left(\frac{x}{2}\right) = +\sqrt{\frac{1-\cos(x)}{2}} = \sqrt{\frac{1-5/13}{2}}$

b) $\cos\left(\frac{x}{2}\right) = -\sqrt{\frac{1+\cos(x)}{2}} = \sqrt{\frac{1+5/13}{2}}$

c) $\tan\left(\frac{x}{2}\right) = \frac{\sin(x/2)}{\cos(x/2)} = \frac{\sqrt{\frac{1-5/13}{2}}}{-\sqrt{\frac{1+5/13}{2}}}$

Problem A $\quad U_n(x) = \dfrac{\sin((n+1)\cos^{-1}(x))}{\sin(\cos^{-1}(x))}$

Soln: First compute

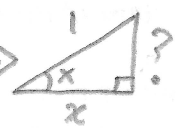
$$\sin(\cos^{-1}(x)).$$

$$\uparrow$$

$$\theta = \cos^{-1}(x)$$

$$\Downarrow$$

$$\cos(\theta) = x \Rightarrow$$



$$\Rightarrow ?^2 + x^2 = 1^2$$

$$\Rightarrow ? = \sqrt{1-x^2}$$

Case $n=1$

$$U_1(x) = \frac{\sin(2\cos^{-1}(x))}{\sin(\cos^{-1}(x))} \underset{\uparrow}{=} \frac{2\cos(\cos^{-1}(x))\cancel{\sin(\cos^{-1}(x))}}{\cancel{\sin(\cos^{-1}(x))}} = 2x$$

dbl angle identity

Case $n=2$

$$U_2(x) = \frac{\sin(3\cos^{-1}(x))}{\sin(\cos^{-1}(x))} \underset{\uparrow}{=} \frac{\sin(\cos^{-1}(x))\cos(2\cos^{-1}(x)) + \cos(\cos^{-1}(x))\sin(2\cos^{-1}(x))}{\sin(\cos^{-1}(x))}$$

$\boxed{\text{use } \sin(\alpha+\beta) \text{ identity on top}}$

$$= \frac{\left(\cancel{\sqrt{1-x^2}}\right)\left[\cos^2(\cos^{-1}(x)) - \sin^2(\cos^{-1}(x))\right] + x\left[\frac{2\cos(\cos^{-1}(x))\cancel{\sin(\cos^{-1}(x))}}{}\right]}{\cancel{\sqrt{1-x^2}}}$$

$$= \left(x^2 - \left(\sqrt{1-x^2}\right)^2\right) + \left(2x^2\right)$$

$$= 3x^2 - 1 + x^2$$

$$= 4x^2 - 1.$$