

§9.1

$$\begin{aligned}\underline{\#5} \quad \sin(x) \cos(x) \sec(x) &= \sin(x) \cancel{\cos(x)} \left(\frac{1}{\cancel{\cos(x)}} \right) \\ &= \sin(x)\end{aligned}$$

$$\begin{aligned}\underline{\#7} \quad \tan(x) \sin(x) + \sec(x) \cos^2(x) &= \frac{\sin(x)}{\cos(x)} \sin(x) + \left(\frac{1}{\cancel{\cos(x)}} \right) \cancel{\cos^2(x)} \\ &= \frac{\sin^2(x)}{\cos(x)} + \frac{\cos^2(x)}{\cos(x)} \\ &= \frac{\sin^2(x) + \cos^2(x)}{\cos(x)} \leftarrow \boxed{\text{top} = 1} \\ &= \frac{1}{\cos(x)}\end{aligned}$$

don't cancel!

$$\underline{\#8} \quad \csc(x) + \cos(x) \cot(-x) = \frac{1}{\sin(x)} + \cos(x) \frac{\cos(-x)}{\sin(-x)}$$

$$\begin{aligned}&\begin{array}{l} \text{Ask} \\ \cos(-x) = \cos(x) \\ \sin(-x) = -\sin(x) \end{array} \rightarrow = \frac{1}{\sin(x)} - \frac{\cos^2(x)}{\sin(x)} \\ &= \frac{1 - \cos^2(x)}{\sin(x)} \leftarrow \begin{array}{l} \text{from } \cos^2(x) + \sin^2(x) = 1 \\ \text{you see} \\ 1 - \cos^2(x) = \sin^2(x) \end{array} \\ &= \frac{\sin^2(x)}{\sin(x)} \\ &= \sin(x)\end{aligned}$$

#13

$$\frac{1 + \tan^2(\theta)}{\csc^2(\theta)} + \sin^2(\theta) + \frac{1}{\sec^2(\theta)} = 1 + \frac{\sin^2 \theta}{\cos^2 \theta} + \sin^2(\theta) + \frac{1}{\cos^2(\theta)}$$

$$\begin{aligned} &= \left(1 + \frac{\sin^2 \theta}{\cos^2 \theta}\right) \sin^2(\theta) + \underbrace{\sin^2(\theta) + \cos^2(\theta)}_{=1} \\ &= \left[\frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} \right] \sin^2(\theta) \\ &= \frac{1}{\cos^2 \theta} \sin^2(\theta) \end{aligned}$$

$$= \frac{\sin^2(\theta)}{\cos^2(\theta)} + 1$$

$$= \frac{\sin^2(\theta) + \cos^2(\theta)}{\cos^2(\theta)}$$

$$= \frac{1}{\cos^2(\theta)}$$

$$= \sec^2(\theta)$$

#16

$$\frac{\tan(x) + \cot(x)}{\csc(x)} = \frac{\frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{\sin(x)}}{\frac{1}{\sin(x)}} = \frac{\frac{\sin^2(x) + \cos^2(x)}{\cos(x)\sin(x)}}{\frac{1}{\sin(x)}}$$

$$= \left(\frac{1}{\cos(x)\sin(x)} \right) \left(\frac{\sin(x)}{1} \right)$$

$$= \frac{1}{\cos(x)}$$

#17

$$\frac{\sec(x) + \csc(x)}{1 + \tan(x)} = \frac{\frac{1}{\cos(x)} + \frac{1}{\sin(x)}}{1 + \frac{\sin(x)}{\cos(x)}} = \frac{\frac{\sin(x) + \cos(x)}{\cos(x)\sin(x)}}{\frac{\cos(x) + \sin(x)}{\cos(x)}}$$

$$= \left(\frac{\sin(x) + \cos(x)}{\cos(x)\sin(x)} \right) \left(\frac{\cos(x)}{\cos(x) + \sin(x)} \right)$$

$$= \frac{1}{\sin(x)}$$

#32 | Verify the identity

$$(\sin(x) + \cos(x))^2 = 1 + 2\sin(x)\cos(x)$$

Solu: Start with left:

$$(\sin(x) + \cos(x))^2 = \sin^2(x) + 2\sin(x)\cos(x) + \cos^2(x)$$

(Note: In the original image, blue arrows point from the $\sin^2(x)$ and $\cos^2(x)$ terms to a circled "=1", indicating the Pythagorean identity.)

use

$$= 1 + 2\sin(x)\cos(x),$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

as was to be shown.

