§9.1

#5 $\sin(x)\cos(x)\sec(x) = \sin(x)\cancel{\cos(x)}\left(\frac{1}{\cancel{\cos(x)}}\right)$

$= \sin(x)$

#7 $\tan(x)\sin(x) + \sec(x)\cos^2(x) = \frac{\sin(x)}{\cos(x)}\sin(x) + \left(\frac{1}{\cancel{\cos x}}\right)\cos\cancel{{}^2}(x)$

don't cancel!

$= \frac{\sin^2(x)}{\cos(x)} + \frac{\cos^2(x)}{\cos(x)}$

$= \frac{\sin^2(x)+\cos^2 x}{\cos x}$ ← top = 1

$= \frac{1}{\cos x}$

#8 $\csc(x) + \cos(x)\cot(-x) = \frac{1}{\sin x} + \cos(x)\frac{\cos(-x)}{\sin(-x)}$

Recall: $\cos(-x) = \cos(x)$, $\sin(-x) = -\sin(x)$

$= \frac{1}{\sin(x)} \cdot \frac{\cos^2(x)}{\sin(x)}$

$= \frac{1-\cos^2(x)}{\sin(x)}$ ← from $\cos^2(x)+\sin^2(x) = 1$ you see $1-\cos^2(x) = \sin^2(x)$

$= \frac{\sin^2(x)}{\sin(x)}$

$= \sin(x)$

#13 $$\frac{1+\tan^2(\theta)}{\csc^2(\theta)} + \sin^2(\theta) + \frac{1}{\sec^2(\theta)} = \frac{1+\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\sin^2\theta}} + \sin^2(\theta) + \frac{1}{\frac{1}{\cos^2(\theta)}}$$

$$= \left(1+\frac{\sin^2\theta}{\cos^2\theta}\right)\sin^2(\theta) + \underbrace{\sin^2(\theta) + \cos^2(\theta)}_{=1}$$

$$\left[\begin{array}{l} = \frac{\cos^2\theta + \sin^2(\theta)}{\cos^2\theta} \\ = \frac{1}{\cos^2\theta} \end{array}\right]$$

$$= \frac{\sin^2(\theta)}{\cos^2(\theta)} + 1$$

$$= \frac{\sin^2(\theta) + \cos^2(\theta)}{\cos^2(\theta)}$$

$$= \frac{1}{\cos^2(\theta)}$$

$$= \sec^2(\theta)$$

#16 $$\frac{\tan(x)+\cot(x)}{\csc(x)} = \frac{\frac{\sin(x)}{\cos(x)} + \frac{\cos(x)}{\sin(x)}}{\frac{1}{\sin(x)}} = \frac{\frac{\sin^2(x)+\cos^2(x)}{\cos(x)\sin(x)}}{\frac{1}{\sin(x)}}$$

$$= \left(\frac{1}{\cos(x)\sin(x)}\right)\left(\frac{\sin(x)}{1}\right)$$

$$= \frac{1}{\cos(x)}$$

#17 $$\frac{\sec(x)+\csc(x)}{1+\tan(x)} = \frac{\frac{1}{\cos(x)} + \frac{1}{\sin(x)}}{1+\frac{\sin(x)}{\cos(x)}} = \frac{\frac{\sin(x)+\cos(x)}{\cos(x)\sin(x)}}{\frac{\cos(x)+\sin(x)}{\cos(x)}}$$

$$= \left(\frac{\sin(x)+\cos(x)}{\cos(x)\sin(x)}\right)\left(\frac{\cos(x)}{\cos(x)+\sin(x)}\right)$$

$$= \frac{1}{\sin(x)}$$

#32 Verify the identity

$$(\sin(x)+\cos(x))^2 = 1+2\sin(x)\cos(x)$$

Soln: Start with left:

$$(\sin(x)+\cos(x))^2 = \sin^2(x)+2\sin(x)\cos(x)+\cos^2(x)$$

$\sin^2(x)+\cos^2(x) = 1$

use

$$= 1+2\sin(x)\cos(x),$$

$(a+b)^2 = a^2+2ab+b^2$

as was to be shown.