

# HW 10 (MATH 1540)

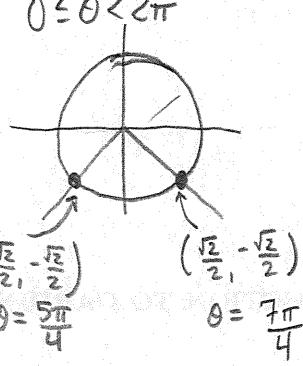
§9.5 #4 Solve  $2\sin(\theta) = -\sqrt{2}$  for  $0 \leq \theta < 2\pi$

$$\downarrow$$

$$\sin(\theta) = -\frac{\sqrt{2}}{2}$$

$$\downarrow$$

$$\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$$



Solution

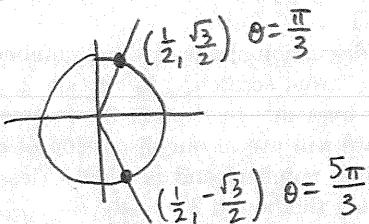
#6 Solve  $2\cos(\theta) = 1$  for  $0 \leq \theta < 2\pi$

$$\downarrow$$

$$\cos(\theta) = \frac{1}{2}$$

$$\downarrow$$

$$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$$



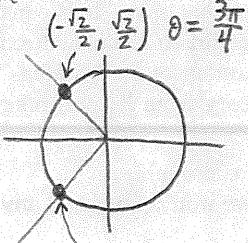
#7 Solve  $2\cos(\theta) = -\sqrt{2}$  for  $0 \leq \theta < 2\pi$

$$\downarrow$$

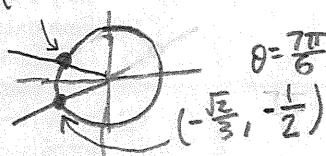
$$\cos(\theta) = -\frac{\sqrt{2}}{2}$$

$$\downarrow$$

$$\theta = \frac{3\pi}{4}, \frac{5\pi}{4}$$



$$(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}) \theta = \frac{5\pi}{4} \quad (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}) \theta = \frac{3\pi}{4}$$



#19 Solve  $2\cos(3\theta) = -\sqrt{2}$  for  $0 \leq \theta < 2\pi$

$$\downarrow$$

$$\cos(3\theta) = -\frac{\sqrt{2}}{3} \Rightarrow \text{Let } 4 = 3\theta, \text{ so solve } \cos(4) = -\frac{\sqrt{2}}{3}$$

↓ General solution

$$\left\{ \begin{array}{l} 4 = \frac{5\pi}{6} + 2n\pi \\ 4 = \frac{7\pi}{6} + 2n\pi \end{array} \right.$$

#19 cont) Which n yield a proper soln?

$$0 \leq \theta < 2\pi \leftrightarrow 0 \leq \theta < \frac{7\pi}{18}$$

$$\begin{cases} 3\theta = 4 = \frac{5\pi}{6} + 2n\pi \\ 3\theta = 4 = \frac{7\pi}{6} + 2n\pi \end{cases} \Rightarrow \begin{cases} \theta = \frac{5\pi}{18} + \frac{2n\pi}{3} = \frac{5\pi}{18} + \frac{12n\pi}{18} \text{ (i)} \\ \theta = \frac{7\pi}{18} + \frac{2n\pi}{3} = \frac{7\pi}{18} + \frac{12n\pi}{18} \text{ (ii)} \end{cases}$$

From (i):

$n=3: \theta = \frac{5\pi}{18} + \frac{36\pi}{18} = \frac{41\pi}{18}$	X
$n=2: \theta = \frac{5\pi}{18} + \frac{24\pi}{18} = \frac{29\pi}{18}$	✓
$n=1: \theta = \frac{5\pi}{18} + \frac{12\pi}{18} = \frac{17\pi}{18}$	✓
$n=0: \theta = \frac{5\pi}{18}$	✓
$n=-1: \theta = \frac{5\pi}{18} - \frac{12\pi}{18} = -\frac{7\pi}{18}$	X

Good solns

All solutions  $0 \leq \theta < 2\pi$   
are these six

From (ii):

$n=3: \frac{7\pi}{18} + \frac{36\pi}{18} = \frac{43\pi}{18}$	X
$n=2: \frac{7\pi}{18} + \frac{24\pi}{18} = \frac{31\pi}{18}$	✓
$n=1: \frac{7\pi}{18} + \frac{12\pi}{18} = \frac{19\pi}{18}$	✓
$n=0: \frac{7\pi}{18} + 0 = \frac{7\pi}{18}$	✓
$n=-1: \frac{7\pi}{18} - \frac{12\pi}{18} = -\frac{5\pi}{18}$	X

Good solns

Solve for  $0 \leq x < 2\pi$

#24  $\tan(x) - 2\sin(x)\tan(x) = 0$

Soln: Factor out  $\tan(x)$  to get

$$\tan(x)(1 - 2\sin(x)) = 0$$

$$\tan(x) = 0$$

$$\Downarrow$$

$$x = 0, \pi$$

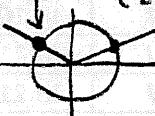
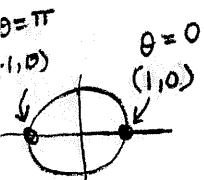
$$(-\frac{\sqrt{3}}{2}, \frac{1}{2}) \theta = \frac{5\pi}{6}$$

$$(\frac{\sqrt{3}}{2}, \frac{1}{2}) \theta = \frac{\pi}{6}$$

$$1 - 2\sin(x) = 0$$

$$\sin(x) = \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}$$



#36 Solve for  $0 \leq t < 2\pi$ :  $6\sin(2t) + 9\sin(t) = 0$

Soln: Use double angle identity

$$\sin(2\theta) = 2\cos(\theta)\sin(\theta)$$

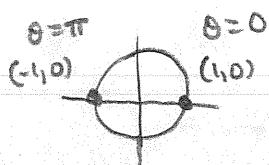
to write

$$6(2\sin(t)\cos(t)) + 9\sin(t) = 0$$

$$12\sin(t)\cos(t) + 9\sin(t) = 0$$

$\downarrow$  factor out  $\sin(t)$

$$\sin(t)[12\cos(t) + 9] = 0$$



$$\sin(t) = 0$$

$$\boxed{t = 0, \pi}$$

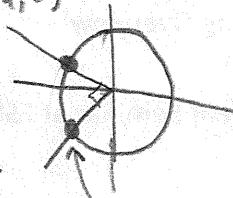
$$12\cos(t) + 9 = 0$$

$$12\cos(t) = -9$$

$$\cos(t) = -\frac{9}{12} = -\frac{3}{4}$$

$$\boxed{t = 2.418, 3.86}$$

$$(-\frac{3}{4}, ?)$$



$$(-\frac{3}{4}, ?)$$

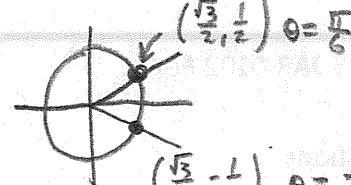
$$\theta = 2\pi - 2.418 \text{ rad} \\ \approx 3.86 \text{ rad}$$

Problem A: General solution of  $\cos(t) = -\frac{\sqrt{3}}{2}$

$\Downarrow$

$$t = \frac{\pi}{6} + 2n\pi$$

$$t = -\frac{\pi}{6} + 2n\pi$$



$$(\frac{\sqrt{3}}{2}, \frac{1}{2}) \theta = \frac{\pi}{6}$$

$$(\frac{\sqrt{3}}{2}, -\frac{1}{2}) \theta = -\frac{\pi}{6}$$

$$\theta = \frac{11\pi}{6}$$

} either ok

where  $n$  is an integer