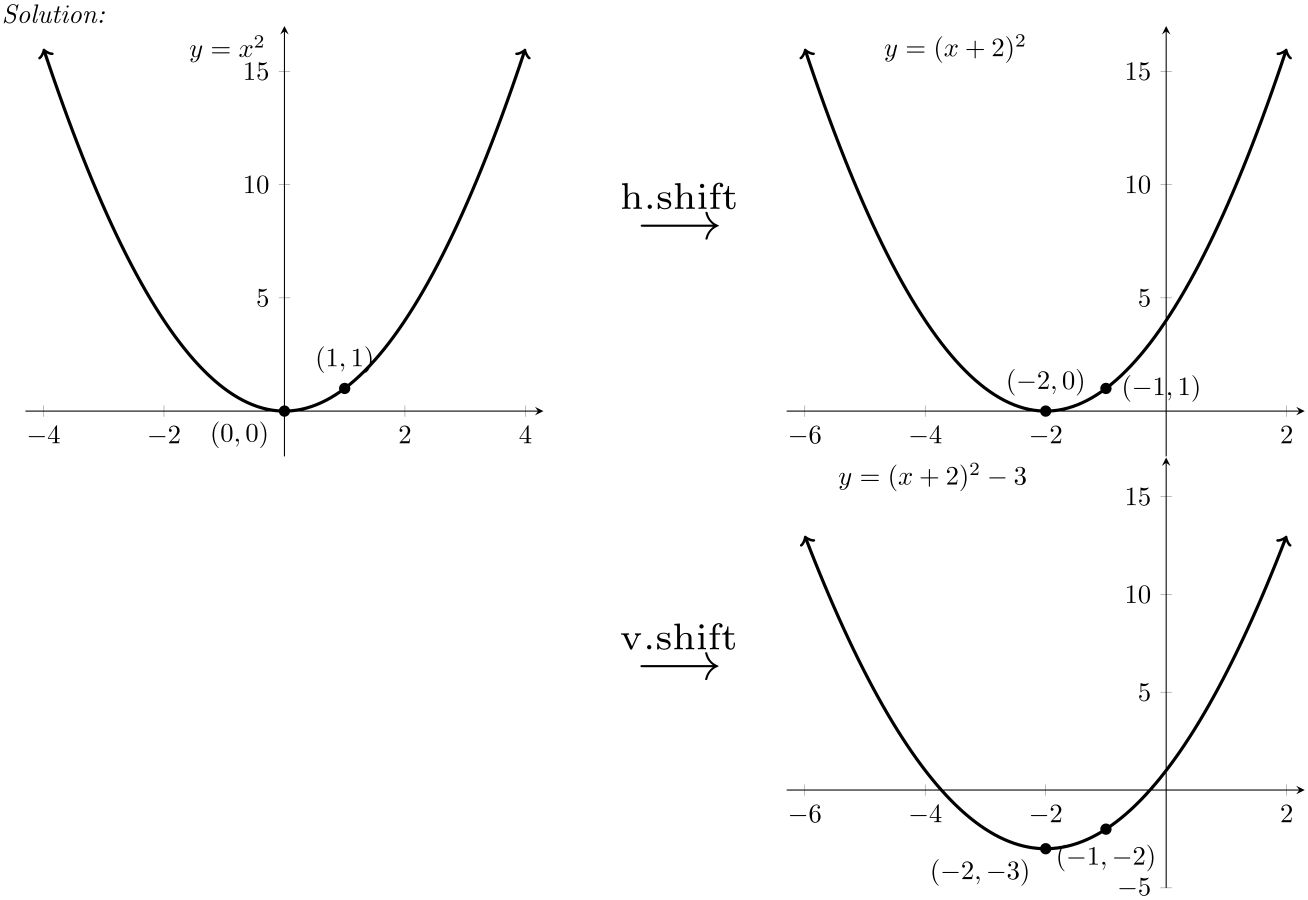
MATH 1540 - EXAM 1 - SPRING 2018
SOLUTION

8 February 2018
Instructor: Tom Cuchta

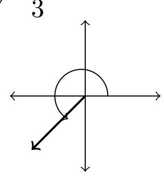
Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (7 points) Sketch a graph of the equation $y = (x+2)^2 - 3$. Include at last two labeled points in your plot to guarantee full credit.
Solution:

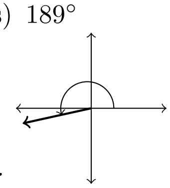


2. (6 points) Draw the specified angle.
   (a) (2 points) $\frac{4\pi}{3}$ radians



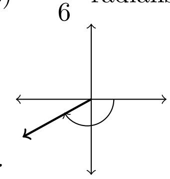
Solution:

   (b) (2 points) 189°



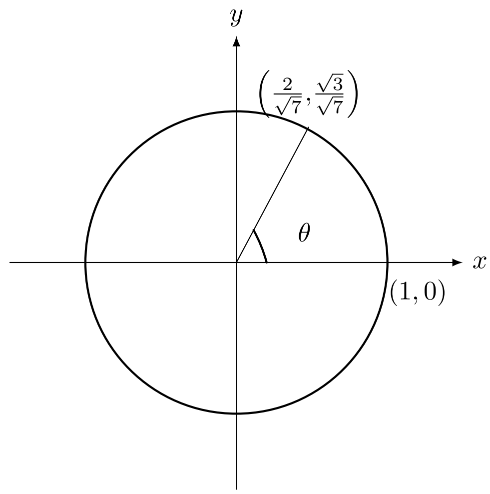
Solution:

   (c) (2 points) $-\frac{5\pi}{6}$ radians



Solution:

3. (6 points) Here is a portion of the unit circle, partially labeled:



(a) (2 points) What is $\sin(\theta)$?

Solution: $\sin(\theta) = \dfrac{\sqrt{3}}{\sqrt{7}}$

(b) (2 points) What is $\tan(\theta)$?

Solution: $\tan(\theta) = \dfrac{\frac{\sqrt{3}}{\sqrt{7}}}{\frac{2}{\sqrt{7}}}$

(c) (2 points) What is $\sec(\theta)$?

Solution: $\sec(\theta) = \dfrac{\sqrt{7}}{2}$

4. (12 points) (a) (4 points) Convert $43°$ to radians.

Solution: Calculate

$$43° = (43°)\left(\frac{2\pi \text{ radians}}{360°}\right) = \frac{2(43)\pi}{360} \text{ radians} = \frac{43}{180} \text{ radians}.$$

(b) (4 points) Convert $\dfrac{3\pi}{11}$ radians to degrees.

Solution: Calculate

$$\frac{3\pi}{11} \text{ radians} = \left(\frac{3\pi}{11} \text{ radians}\right)\left(\frac{360°}{2\pi \text{ radians}}\right) = \left(\frac{3(360)}{2(11)}\right)^{\circ} = \left(\frac{540}{11}\right)^{\circ}.$$

(c) (4 points) Convert 4 radians to degrees.

Solution: Calculate

$$4 \text{ radians} = (4 \text{ radians})\left(\frac{360°}{2\pi \text{ radians}}\right) = \left(\frac{180}{\pi}\right)^{\circ}.$$

5. (12 points) Find an exact value for...

(a) (4 points) $\cos\left(\dfrac{\pi}{6}\right)$

Solution: Since the coordinates of the angle $\dfrac{\pi}{6}$ on the unit circle are $\left(\dfrac{\sqrt{3}}{2}, \dfrac{1}{2}\right)$, we see that

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}.$$

(b) (4 points) $\cot\left(\frac{\pi}{2}\right)$

Solution: Since the coordinates of the angle $\frac{\pi}{2}$ on the unit circle are $(0, 1)$, we see that

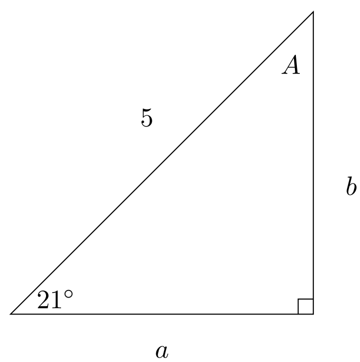
$$\cot\left(\frac{\pi}{2}\right) = \frac{0}{1} = 0.$$

(c) (4 points) $\sin\left(\frac{11\pi}{6}\right)$

Solution: Since the coordinates of $\frac{11\pi}{6}$ on the unit circle are $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$, we see that

$$\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}.$$

6. (6 points) Solve the triangle.



Solution:
<u>Find A</u>
We use the fact that $21° + \underbrace{90°}_{\text{right angle}} + A = 180°$. Solving for A yields

$$A = 180° - 90° - 21° = 90° - 21° = 69°.$$

<u>Find b</u>
We use the sine function to write $\sin(21°) = \frac{b}{5}$. Therefore solving for b yields

$$b = 5\sin(21°).$$

<u>Find a</u>
We use the cosine function to write $\cos(21°) = \frac{a}{5}$. Therefore solving for a yields

$$a = 5\cos(21°).$$

7. (8 points) (a) (4 points) Solve the equation $3x + 4 = 9$.

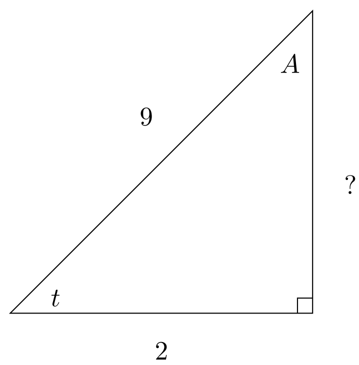
Solution: Subtract 4 to get $3x = 9 - 4 = 5$. Divide by 5 to get $x = \frac{5}{3}$.

(b) (4 points) Solve the equation $x^2 + x - 6 = 0$.

Solution: Factor the left-hand side to get $(x + 3)(x - 2) = 0$. Using the zero product property of the real numbers, we may conclude that either $x+3 = 0$ or $x-2 = 0$. Thus either $x = -3$ or $x = 2$.

8. (15 points) If $\cos(t) = \dfrac{2}{9}$ and t is in quadrant IV, find the other five trigonometric functions.

Solution: Since we are told that $\cos(t) = \dfrac{2}{9}$, we draw a triangle in which that occurs:



To find the other trigonometric functions of t, we must find the side labelled ?:

Find ?

Using the Pythagorean theorem, we write $2^2 + ?^2 = 9^2$, so we get $4 + ?^2 = 81$. Subtract 4 and get $?^2 = 77$ and take the square root to get $? = \pm\sqrt{77}$. We must throw away the negative solution because it is physically meaningless (negative side length in a triangle doesn't make sense). Therefore $? = \sqrt{77}$.

Also we are told that t is in quadrant IV. This means that sine, tangent, cosecant, and cotangent will be negative while cosine and second will be positive. Now we may answer the question.

The other five trig functions

$$\sin(t) = -\frac{?}{9} = -\frac{\sqrt{77}}{9},$$

$$\tan(t) = -\frac{?}{2} = -\frac{\sqrt{77}}{2},$$

$$\sec(t) = \frac{9}{2},$$

$$\csc(t) = -\frac{9}{?} = -\frac{9}{\sqrt{77}},$$

and

$$\cot(t) = -\frac{2}{?} = -\frac{2}{\sqrt{77}}.$$

9. (8 points) Find the radius a circle must have if an arc length of 7 is subtended by an angle of $\theta = 12°$. Express your final answer accurate to at least two decimal places.
Solution: We want to use the formula $s = r\theta$, but to do so, θ must be expressed in radians. So first we must convert θ to radians:

$$12° = (12°)\left(\frac{2\pi \text{ radians}}{360°}\right) = \frac{\pi}{15} \text{ radians.}$$

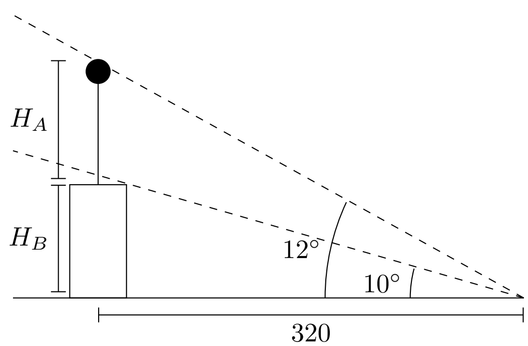
Therefore we can now use the formula $s = r\theta$ with $s = 7$ and $\theta = \frac{\pi}{15}$, and then we can use it to solve for r (as requested). Plugging in:

$$7 = (r)\left(\frac{\pi}{15}\right).$$

To solve for r, divide both sides by $\frac{\pi}{15}$ to get

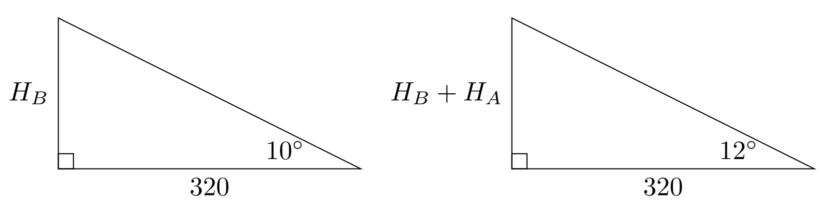
$$r = \frac{7}{\frac{\pi}{15}} = \frac{7}{1}\left(\frac{15}{\pi}\right) = \frac{105}{\pi} \approx 33.42.$$

10. (16 points) There is an antenna on the top of a building. From a location 320 feet from the base of the building, the angle of elevation to the top of the building is measured to be 10°. From the same location, the angle of elevation to the top of the antenna is measured to be 12°. Find the height of the antenna. Express your final answer accurate to at least two decimal places.
Solution: We start with a figure to describe the problem:



Thus, we seek H_A.

Note that we have *two* right triangles in this picture, reproduced (in a simplified form) below:



Thus,

$$\tan(10°) = \frac{H_B}{320}$$
$$H_B = 320\tan(10°)$$

and

$$\tan(12°) = \frac{H_A + H_B}{320}$$
$$H_A + H_B = 320\tan(12°)$$

So we see

$$\begin{aligned} H_A &= (H_A + H_B) - H_B \\ &= 320\tan(12^\circ) - 320\tan(10^\circ) \end{aligned}$$

$$\boxed{a \approx 11.59 \text{ feet.}}$$

11. (4 points) Use your calculator to compute the following trig functions accurate to at least two decimal places.
    (a) (1 point) $\sin(21^\circ)$
    Solution: Calculate
    $$\sin(21^\circ) \approx 0.35836.$$
    (b) (1 point) $\cos\left(\frac{9\pi}{12}\right)$
    Solution: Calculate
    $$\cos\left(\frac{9}{\pi}\right) \approx -0.96193.$$
    (c) (1 point) $\sec(11^\circ)$
    Solution: Calculate
    $$\sec(11^\circ) = \frac{1}{\cos(11^\circ)} \approx 1.01871.$$
    (d) (1 point) $\sin(21)$
    Solution: Calculate
    $$\sin(21) \approx 0.83665.$$