HW 10 MATH 1540 FALL 2018

§9.2

#11 $\sin\left(\overset{\alpha}{x} - \overset{\beta}{\frac{3\pi}{4}}\right) \overset{\text{difference identity}}{=} \sin(x)\cos\left(\frac{3\pi}{4}\right) - \cos(x)\sin\left(\frac{3\pi}{4}\right)$

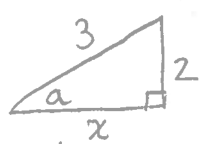
$= \left(-\frac{\sqrt{2}}{2}\right)\sin(x) - \frac{\sqrt{2}}{2}\cos(x)$

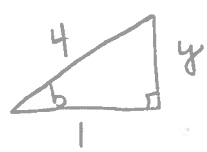
#16 $\cot\left(\frac{\pi}{2} - x\right) = \dfrac{\cos(\pi/2 - x)}{\sin(\pi/2 - x)} = \dfrac{\underbrace{\cos(\pi/2)}_{=0}\cos(x) + \overbrace{\sin(\pi/2)}^{=1}\sin(x)}{\underbrace{\sin(\pi/2)}_{=1}\cos(x) - \underbrace{\cos(\pi/2)}_{=0}\sin(x)}$

$= \dfrac{\sin(x)}{\cos(x)} = \tan(x)$

#18 $\sin(2x)\cos(5x) - \sin(5x)\cos(2x) \overset{\text{difference identity}}{=} \sin(2x - 5x) = \sin(-3x)$

#20 Given: $\sin(a) = \frac{2}{3}$, $\cos(b) = -\frac{1}{4}$





$x^2 + 2^2 = 3^2$

$x^2 = 9 - 4$

$x = \sqrt{5}$

$\Rightarrow \cos(a) = -\frac{\sqrt{5}}{3}$

minus b/c a is in QII

$1^2 + y^2 = 4^2$

$y^2 = 16 - 1$

$y = \sqrt{15}$

$\Rightarrow \sin(b) = \frac{\sqrt{15}}{4}$

Now calculate $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

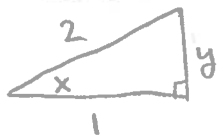
$= \left(\frac{2}{3}\right)\left(-\frac{1}{4}\right) + \left(-\frac{\sqrt{5}}{3}\right)\left(\frac{\sqrt{15}}{4}\right) = \frac{-3 - \sqrt{75}}{12}$

and $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$

$= \left(-\frac{\sqrt{5}}{3}\right)\left(-\frac{1}{4}\right) - \left(\frac{2}{3}\right)\left(\frac{\sqrt{15}}{4}\right) = \frac{\sqrt{5} - 2\sqrt{15}}{12}$

§9.3 | #7 | Given $\cos(x) = -\frac{1}{2}$, x in QIII

$\Downarrow$



$y \rightarrow 1^2 + y^2 = 2^2 \Rightarrow \sin(x) = -\frac{\sqrt{3}}{2}$ (minus b/c x in QIII)

$y^2 = 4 - 1$

$y = \sqrt{3}$

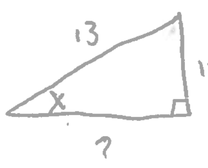
Calculate

$$\sin(2x) = 2\sin(x)\cos(x)$$
$$= \cancel{2}\left(-\frac{\sqrt{3}}{\cancel{2}}\right)\left(-\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$
$$= \left(-\frac{1}{2}\right)^2 - \left(-\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{1-3}{4} = -\frac{1}{2}$$

$$\tan(2x) = \frac{\sin(2x)}{\cos(2x)} = \frac{\sqrt{3}/2}{-1/2} = -\sqrt{3}$$

#21 | Given: $\sin(x) = -\frac{12}{13}$, x in QIII $\longrightarrow$ $\frac{x}{2}$ in QII



$12 \rightarrow ?^2 + 12^2 = 13^2 \Rightarrow \cos(x) = -\frac{5}{13}$ (x in QIII)

$?^2 = 169 - 144$

$? = \sqrt{25} = 5$

Compute

(+ b/c x in QII)

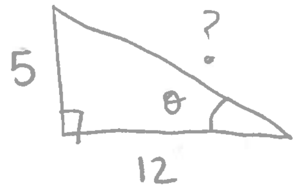
$$\sin\left(\frac{x}{2}\right) = +\sqrt{\frac{1-\cos(x)}{2}} = \sqrt{\frac{1-(-5/13)}{2}} = \sqrt{\frac{1+5/13}{2}}$$

(− b/c $\frac{x}{2}$ in QII)

$$\cos\left(\frac{x}{2}\right) = -\sqrt{\frac{1+\cos(x)}{2}} = \sqrt{\frac{1+(-5/13)}{2}} = \sqrt{\frac{1-5/13}{2}}$$

$$\tan\left(\frac{x}{2}\right) = \frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)} = \frac{\sqrt{\frac{1+5/13}{2}}}{\sqrt{\frac{1-5/13}{2}}}$$

#24 Given:



$\rightarrow 12^2 + 5^2 = ?^2$

$144 + 25 = ?^2$

$? = \sqrt{169} = 13$

$\rightarrow \sin(\theta) = \frac{5}{13}$ 3

$\cos(\theta) = \frac{12}{13}$

Calculate

$\sin(2\theta) = 2\sin(\theta)\cos(\theta) = 2\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{2(5/13)(12/13)}{(12/13)^2 - (5/13)^2}$