MATH 1540 – EXAM 1 FALL 2018
SOLUTION

14 September 2018
Instructor: Tom Cuchta

Instructions:

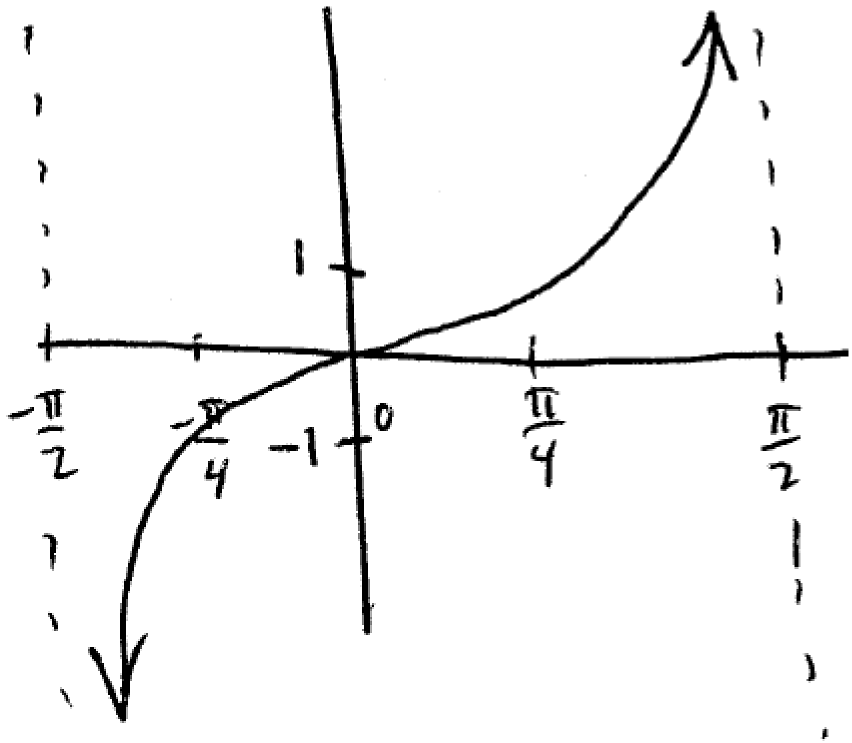
- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (18 points) (a) (6 points) Sketch a graph of $y = \tan(x)$.

Solution (this problem resembles **#23 in HW4**): Take the anchor points

$$-\frac{\pi}{2} \quad -\frac{\pi}{4} \quad 0 \quad \frac{\pi}{4} \quad \frac{\pi}{2}$$

Now plot:



(b) (12 points) Sketch the graph of $y = 2\sin\left(3\left(x - \frac{\pi}{4}\right)\right) + 5$.

Solution (this problem resembles **#10, #18, #20 in HW4**): Begin by identifying the function transformations involved. The first is a horizontal shift to the right by $\frac{\pi}{4}$, the second is a horizontal compression which divides x-values by 3, the third is a vertical stretch which multiplies y-values by 2, and the final is the vertical shift up by 5.

Now start with the anchor points

$$0 \quad \frac{\pi}{2} \quad \pi \quad \frac{3\pi}{2} \quad 2\pi.$$

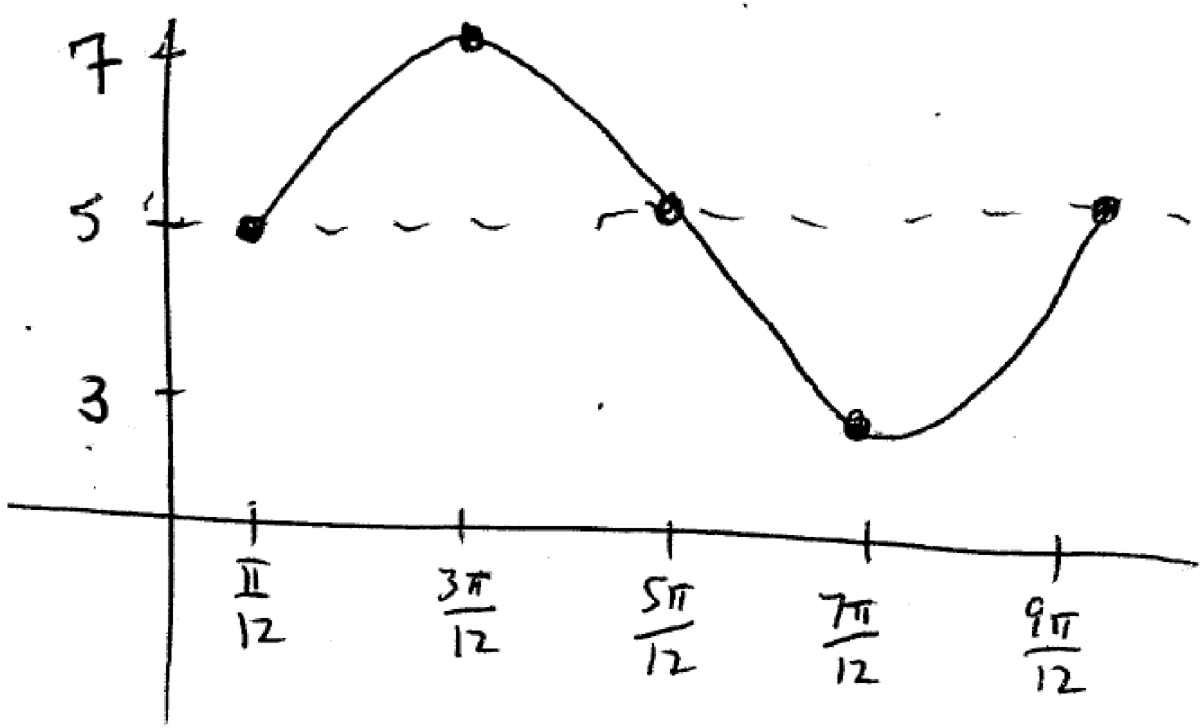
The horizontal shift transforms the anchor points into

$$\frac{\pi}{4} \quad \frac{3\pi}{4} \quad \frac{5\pi}{4} \quad \frac{7\pi}{4} \quad \frac{9\pi}{4}.$$

The horizontal compression transforms the anchor points into

$$\frac{\pi}{12} \quad \frac{3\pi}{12} \quad \frac{5\pi}{12} \quad \frac{7\pi}{12} \quad \frac{9\pi}{12}.$$

The vertical values $-1, 0, 1$ when multiplied by 2 becomes $-2, 0, 2$ and when 5 is added to them becomes $3, 5, 7$. With this information we now plot:



2. (12 points) (a) (4 points) Convert $19°$ to radians.

Solution (this problem resembles **Quiz 1 and #34 in HW1**): Compute

$$19° = \left(19°\right)\left(\frac{2\pi \text{ rad}}{360°}\right) = \frac{19(2\pi)}{360}\text{rad} = \frac{19\pi}{180}\text{rad}$$

(b) (4 points) Convert $\frac{2\pi}{7}$ radians to degrees.

Solution (this problem resembles **Quiz 1 and #27 in HW1**): Compute

$$\frac{2\pi}{7}\text{rad} = \left(\frac{2\pi}{7}\text{rad}\right)\left(\frac{360°}{2\pi\text{rad}}\right) = \left(\frac{360}{7}\right)^{\circ}$$

(c) (4 points) Convert 9 radians to degrees.

Solution (this problem resembles **Quiz 1 and #27 in HW1**): Compute

$$9\text{rad} = \left(9\text{rad}\right)\left(\frac{360°}{2\pi\text{rad}}\right) = \left(\frac{9(180)}{\pi}\right)^{\circ} = \left(\frac{1620}{\pi}\right)^{\circ}$$

3. (15 points) Find an exact value for...

(a) (5 points) $\cos\left(\frac{\pi}{6}\right)$

Solution (this problem resembles **Quiz 2 and #10, #19 in HW2**): From the unit circle, we see

$$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}.$$

(b) (5 points) $\cot\left(\frac{\pi}{2}\right)$

Solution (this problem resembles **Quiz 2 and #10, #19 in HW2**): From the unit circle, we see

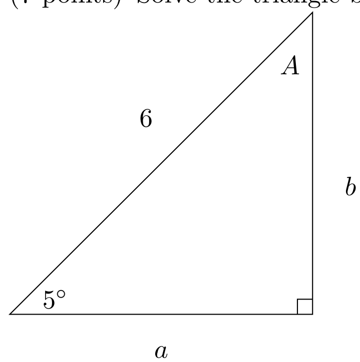
$$\cot\left(\frac{\pi}{2}\right) = \frac{0}{1} = 0.$$

(c) (5 points) $\sin\left(\frac{11\pi}{6}\right)$

Solution (this problem resembles **Quiz 2 and #10, #19 in HW2**): From the unit circle, we see

$$\sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}.$$

4. (7 points) Solve the triangle by finding all unknown sides and unknown angles.



Solution (this problem resembles **#30 in HW2**): First note that

$$\sin(5^\circ) = \frac{b}{6},$$

and solve this for b by multiplying by 6 to get

$$b = 6\sin(5^\circ).$$

Now note that

$$\cos(5^\circ) = \frac{a}{6},$$

and solve this for a by multiplying by 6 to get

$$a = 6\cos(5^\circ).$$

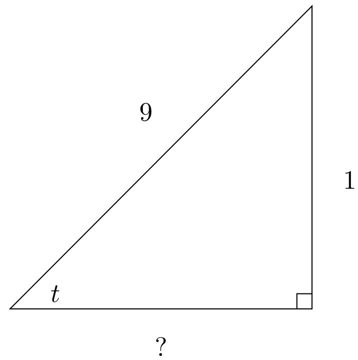
Finally find A by noting that

$$5^\circ + \underbrace{90^\circ}_{\text{the right angle}} + A = 180^\circ.$$

Subtracting 5° and 90° yields

$$A = 180^\circ - 90^\circ - 5^\circ = 85^\circ.$$

5. (15 points) If $\sin(t) = -\frac{1}{9}$ and t is in quadrant III, then find the other five trigonometric functions.
Solution (this problem resembles **#52 in HW3**):
First draw a triangle for which $\sin(t) = \frac{1}{9}$:



First find ? by using the Pythagorean theorem:

$$?^2 + 1^2 = 9^2$$

yields

$$?^2 = 81 - 1$$

yields

$$?^2 = 80,$$

hence

$$? = \pm\sqrt{80},$$

but we throw out the negative solution for physical reasons to get

$$? = \sqrt{80}.$$

From this, we may compute

$$\begin{array}{ll} \sin(t) = -\frac{1}{9} & \csc(t) = -9 \\ \cos(t) = -\frac{\sqrt{80}}{9} & \sec(t) = -\frac{9}{\sqrt{80}} \\ \tan(t) = \frac{1}{\sqrt{80}} & \cot(t) = \sqrt{80} \end{array}$$

6. (8 points) Find the radius a circle must have if an arc length of 2 is subtended by an angle of $\theta = 15°$. Express your final answer accurate to at least two decimal places.
Solution (this problem resembles **#24 and #43 in HW1**): We will use the formula $s = r\theta$. First note that $s = 2$ in that formula. We are given $\theta = 15°$ so we must first convert it into radians:

$$15° = \left(15°\right)\left(\frac{2\pi\text{rad}}{360°}\right) = \frac{30\pi}{360}\text{rad} = \frac{\pi}{12}\text{rad}$$

So now plugging into $s = r\theta$ yields

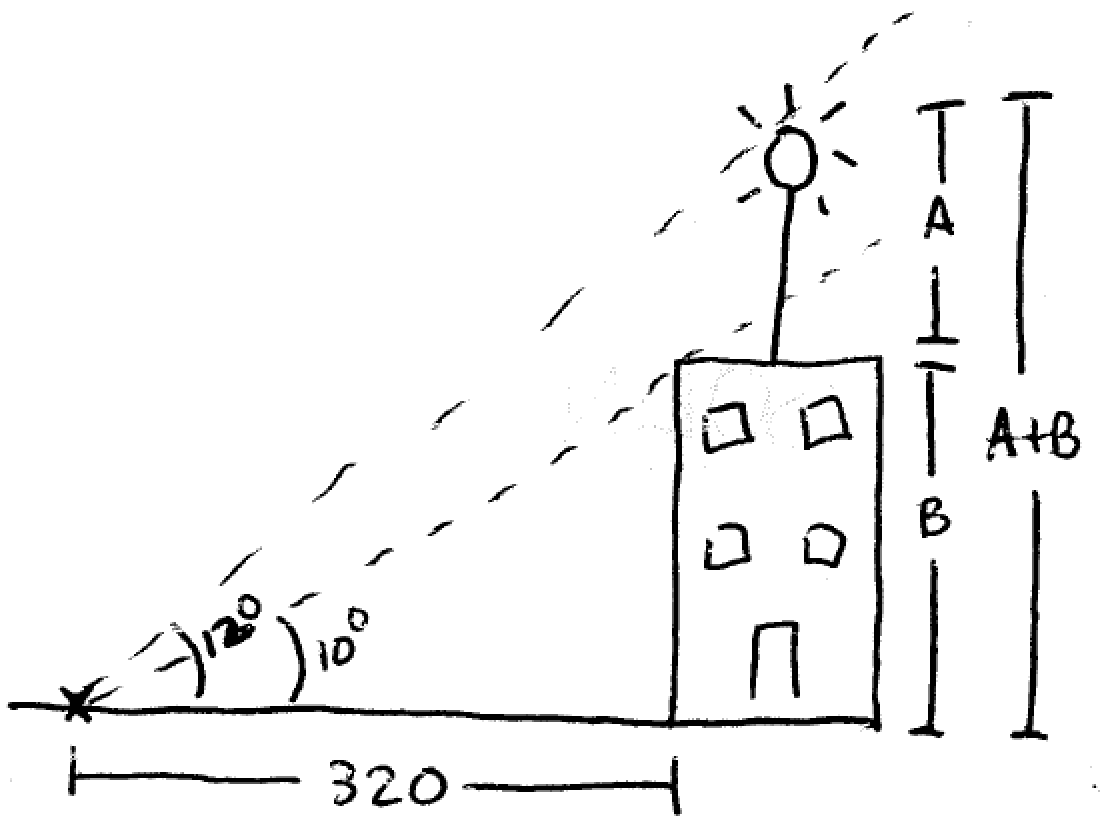
$$2 = r\left(\frac{\pi}{12}\right).$$

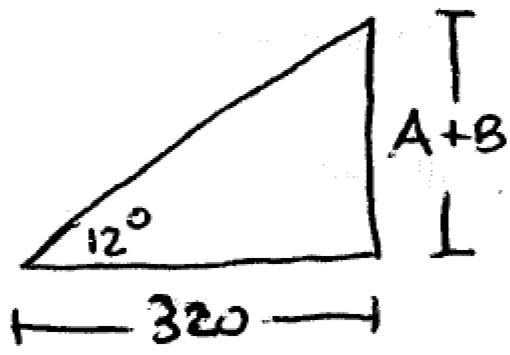
Solving this for r by multiplying by 12 and dividing by π yields

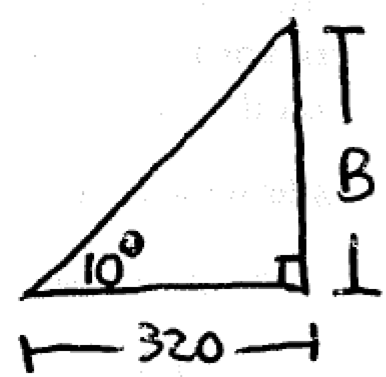
$$r = \frac{24}{\pi} \approx 7.639.$$

7. (14 points) There is an antenna on the top of a building. From a location 320 feet from the base of the building, the angle of elevation to the top of the building is measured to be 10°. From the same location, the angle of elevation to the top of the antenna is measured to be 12°.

   (a) (5 points) Draw a picture of this scenario.
   Solution (this problem resembles **#48 in HW3**):







(b) (9 points) Find the height of the antenna. Express your final answer accurate to at least two decimal places.
Solution: From the first triangle,

$$\tan(12^\circ) = \frac{A+B}{320},$$

hence

$$A + B = 320\tan(12^\circ).$$

From the second triangle,

$$\tan(10^\circ) = \frac{B}{320},$$

hence

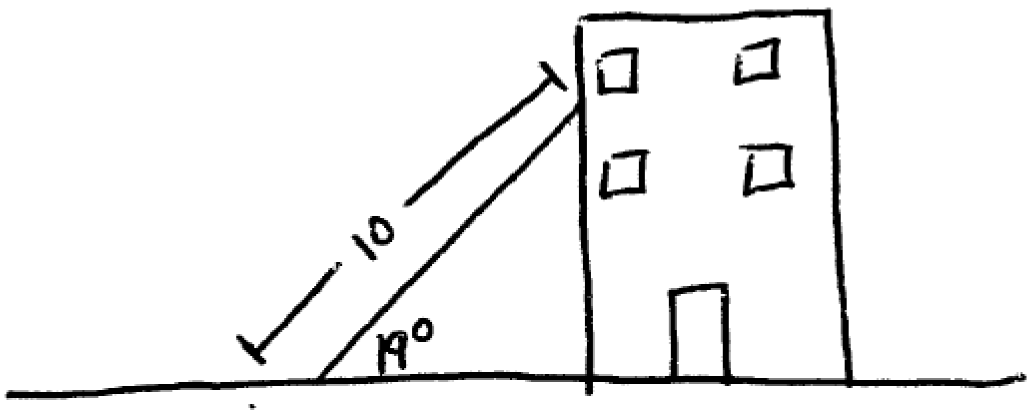
$$B = 320\tan(10^\circ).$$

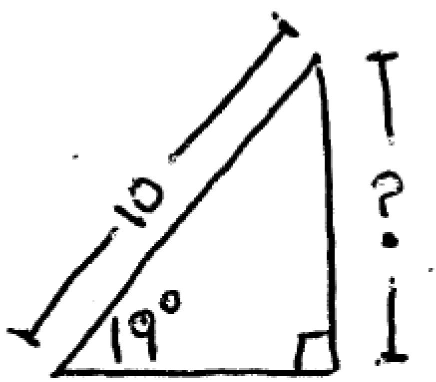
Since A is the height of the antenna, we get

$$A = (A+B) - B = 320\tan(12^\circ) - 320\tan(10^\circ) \approx 11.59 \text{ ft}.$$

8. (11 points) A 10 foot long ladder leans against a building and makes an angle of 19° with the ground.

(a) (5 points) Draw a picture of this scenario.
Solution (this problem resembles **#53 in HW3**):





(b) (6 points) How far up the building does the top of the ladder touch? Express your final answer accurate to at least two decimal places.
Solution: From the triangle, we need to find "?". So, compute

$$\sin(19°) = \frac{?}{10},$$

therefore multiplying by 10 yields

$$? = \sin(19°) \approx 3.255 \text{ ft}.$$