X

Problem A

$$\langle \vec{x}, \vec{y} \rangle = \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ 17 \end{bmatrix}$$

$$= 3(2) + (2)(1) + (-1)(17)$$

$$= 6 + 2 - 17$$

$$= -9$$

Problem B

$$\langle p, q \rangle = \oint \langle x-1, x^2 \rangle$$

$$= \int_0^\infty (x-1)x^2 e^{-x}\, dx$$

$$= 4$$

3 ints by parts or software

2 ints by parts or software

$$\langle p, p \rangle = \langle x-1, x-1 \rangle = \int_0^\infty (x-1)(x-1)e^{-x} dx = 1$$

Problem C

$$\langle f, g \rangle = \langle \log(x), 1 \rangle = \int_0^1 \log(x)\, dx = -1$$

$$\langle h_1, h_2 \rangle = \int_0^1 x^2 \sin(x)\, dx = -2 + 2\sin(1) + \cos(1)$$

Problem D

$$\langle c_k, d_k \rangle = \left\langle \sqrt{\tfrac{1}{k!}}, \sqrt{\tfrac{1}{k!}} \right\rangle = \sum_{k=0}^{\infty} \sqrt{\tfrac{1}{k!}} \sqrt{\tfrac{1}{k!}}$$

$$= \sum \frac{1}{k!}$$

$$= e$$

Problem E $\langle \vec{x}, \vec{y} \rangle = \langle 5+4i, 9-11i \rangle$

$$= (5+4i)\overline{(9-11i)}$$

$$= (5+4i)(9+11i)$$

$$= 45+55i+36i-44$$

$$= 1+91i$$

$$\frac{11-5i}{2+i} = \frac{11-5i}{2+i}\left(\frac{2-i}{2-i}\right)$$

$$= \frac{(11-5i)(2-i)}{4+1}$$

$$= \frac{22-11i-10i+5i^2}{5}$$

$$= \frac{17}{5} - \frac{21}{5}i$$

$$\langle z_1, z_2 \rangle = \left\langle 21+16i, \frac{11-5i}{2+i} \right\rangle$$

$$= \left\langle 21+16i, \frac{17}{5} - \frac{21}{5}i \right\rangle$$

$$= (21+16i)\overline{\left(\frac{17}{5} - \frac{21}{5}i\right)}$$

$$= (21+16i)\left(\frac{17}{5} + \frac{21}{5}i\right)$$

$$= \frac{21}{5} + \frac{713i}{5}$$

Problem F

$$\langle \vec{a}, \vec{b} \rangle = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ -3 \\ 2 \\ 1 \end{bmatrix} = (1)(-4)+(2)(-3)+(3)(2)+(4)(1)$$

$$= -4-6+6+4$$

$$= 0 \quad \text{(orthogonal)}$$

Problem G $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

Show mutually orthogonal:

$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} = 0$ ✓ $\quad \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = 0$ ✓

$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = 0$ ✓ $\quad \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 0$ ✓

$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 0$ ✓ $\quad \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 0$ ✓

$\Rightarrow$ mutually orthogonal

The set $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ is not a mutualy orthogonal set because, e.g.

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} = 1 \neq 0$$

Problem H:

$$\langle 4x^2+3x+9, 1\rangle \overset{\text{linear in 1st arg}}{=} 4\langle x^2, 1\rangle + 3\langle x, 1\rangle + 9\langle 1, 1\rangle$$
$$= 4\left(\frac{\sqrt{\pi}}{2}\right) + 3(0) + 9(\sqrt{\pi})$$
$$= \sqrt{\pi}(2+9)$$
$$= 11\sqrt{\pi}$$

$$\langle 32x^5 - 64x^3 + 24x, 1\rangle = 32\langle x^5, 1\rangle - 64\langle x^3, 1\rangle + 24\langle x, 1\rangle$$
$$= 0 + 0 + 0$$
$$= 0$$

Problem I:

$$\operatorname{proj}_{x^2-3x}(5x+2) = \frac{\langle 5x+2, x^2-3x\rangle}{\langle x^2-3x, x^2-3x\rangle}(x^2-3x)$$
$$= \frac{\int_0^1 (5x+2)(x^2-3x)x^2\,dx}{\int_0^1 (x^2-3x)(x^2-3x)x^2\,dx}(x^2-3x)$$
$$= \frac{-49/15}{33/35}(x^2-3x)$$

$$\text{proj}_{5x+2}(x^2-3x) = \frac{\int_0^1 (x^2-3x)(5x+2)x^2\,dx}{\int_0^1 (5x+2)(5x+2)x^2\,dx}(5x+2)$$

$$= \frac{-49/15}{34/3}(5x+2)$$

<u>Problem J</u> $\left\{\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}2\\4\\8\end{bmatrix}, \begin{bmatrix}3\\9\\27\end{bmatrix}\right\}$

done in class

<u>Problem K</u> Apply Grm-Schmidt to $\{\underset{\vec{v}_1}{1}, \underset{\vec{v}_2}{x}, \underset{\vec{v}_3}{x^2}, \underset{\vec{v}_4}{x^3}\}$

$$\vec{u}_1 = \vec{v}_1 = 1$$

$$\vec{u}_2 = \vec{v}_2 - \text{proj}_{\vec{u}_1}(\vec{v}_2) = x - \frac{\langle x, 1\rangle}{\langle 1, 1\rangle}(1)$$

$$= x - \frac{\int_{-1}^1 x\,dx}{\int_{-1}^1 1\,dx}(1)$$

$$= x - 0 = x$$

$$\vec{u}_3 = \vec{v}_3 - \text{proj}_{\vec{u}_1}(\vec{v}_3) - \text{proj}_{\vec{u}_2}(\vec{v}_3) = x^2 - \frac{\langle x^2, 1\rangle}{\langle 1, 1\rangle}(1) - \frac{\langle x^2, x\rangle}{\langle x, x\rangle}x$$

$$= x^2 - \frac{\int_{-1}^1 x^2\,dx}{\int_{-1}^1 1\,dx} - \frac{\int_{-1}^1 x^3\,dx}{\int_{-1}^1 x^2\,dx}$$

$$= x^2 - \frac{2/3}{2} - 0$$

$$= x^2 - 1/3$$

X

$$\vec{u_4} = \vec{v_4} - \text{proj}_{\vec{u_1}}(\vec{v_4}) - \text{proj}_{\vec{u_2}}(\vec{v_4}) - \text{proj}_{\vec{u_3}}(\vec{v_4})$$

$$= x^3 - \frac{\langle x^3, 1\rangle}{\langle 1,1\rangle}(1) - \frac{\langle x^3, x\rangle}{\langle x, x\rangle}(x) - \frac{\langle x^3, x^2 - 1/3\rangle}{\langle x^2 - 1/3, x^2 - 1/3\rangle}(x^2 - 1/3)$$

$$= x^3 - \frac{\int_{-1}^{1} x^3\,dx}{\int_{-1}^{1} 1\,dx} - \frac{\int_{-1}^{1} x^4\,dx}{\int_{-1}^{1} x^2\,dx}(x) - \frac{\int_{-1}^{1} x^5 - \frac{1}{3}x^3\,dx}{\int_{-1}^{1} (x^2 - 1/3)^2\,dx}(x^2 - 1/3)$$

$$= x^3 - 0 - \frac{2/5}{2/3}(x) - 0$$

$$= x^3 - \frac{3}{5}x$$