§9.1 #18:

$$\frac{n!}{(n+2)!} = \frac{\cancel{n!}}{(n+2)(n+1)\cancel{n!}} = \frac{1}{(n+2)(n+1)}$$

§9.1 #21:

~~Proof by~~ ratio:

$$\lim_{n\to\infty} \frac{5n^2}{n^2+2} = \lim_{n\to\infty} \frac{5n^2}{1n^2+2}$$

$$= \frac{5}{1} = 5$$

Proof by L'Hôpital:

$$\lim_{n\to\infty} \frac{5n^2}{n^2+2} = \lim_{x\to\infty} \frac{5x^2}{x^2+2}$$

$$= \frac{\infty}{\infty} \text{ (indeterminate form)}$$

$$\overset{L.H.}{=} \lim_{x\to\infty} \frac{10\cancel{x}}{2\cancel{x}}$$

$$= \frac{10}{2} = 5$$

Proof by algebra:

$$\lim_{n\to\infty} \frac{5n^2}{n^2+2} = \lim_{n\to\infty} \frac{(5n^2)}{(n^2+2)} \frac{\left(\frac{1}{n^2}\right)}{\left(\frac{1}{n^2}\right)}$$

$$= \lim_{n\to\infty} \frac{5}{1+\underbrace{\frac{2}{n^2}}_{\to 0}}$$

$$= \frac{5}{1} = 5$$

§9.1 #(30) $a_n = 8 + \frac{5}{n}$ converges

because

$$\lim_{n\to\infty} 8 = 8$$

and $\cancel{\lim_{n\to\infty} \frac{5}{n} = 0}$ $\lim_{n\to\infty} \frac{5}{n} = 0,$

so

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left(8 + \frac{5}{n}\right)$$

because both limits exist!

$$= \left(\lim_{n\to\infty} 8\right) + \left(\lim_{n\to\infty} \frac{5}{n}\right)$$

$$= 8 + 0$$

$$= 8$$

§9.1 #31 $a_n = (-1)^n \left(\frac{n}{n+1}\right)$ $\boxed{\text{diverges}}$

because while

$$\lim_{n\to\infty} \frac{n}{n+1} = 1,$$

the limit

$\lim_{n\to\infty} (-1)^n$ does not exist (oscillation)

§9.1 #43

$a_n = \frac{\sin(n)}{n}$ converges

because sine is bounded, i.e.

$\left[\text{for all } \underset{\boxed{\text{real numbers}}}{x}, \quad -1 \leq \sin x \leq 1 \right]$

while the denominator increases without bound, meaning that

$$-\frac{1}{n} \leq \frac{\sin(n)}{n} \leq \frac{1}{n}$$

$$\Downarrow$$

$$\underbrace{\lim_{n\to\infty} -\frac{1}{n}}_{\downarrow \text{ know} \atop 0} \leq \lim_{n\to\infty} \frac{\sin(n)}{n} \leq \underbrace{\lim_{n\to\infty} \frac{1}{n}}_{\downarrow \text{ know} \atop 0}$$

Therefore,

$$0 \leq \lim_{n\to\infty} \frac{\sin(n)}{n} \leq 0$$

$\Downarrow$ squeeze theorem

$$\lim_{n\to\infty} \frac{\sin n}{n} = 0.$$