1. Formulas

By this point these should be **entirely internalized** for you:

$$\int \frac{1}{x} \mathrm{d}x = \ln(|x|) + C \tag{1.1}$$

$$\int e^x \mathrm{d}x = e^x + C \tag{1.2}$$

If you remember how the "fundamental theorem of calculus" tells us how to turn integral formulas into derivative formulas ("derivatives and integrals are inverses"), then you get the following two formulas "for free" from (1.1) and (1.2):

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln(x) = \frac{1}{x} \tag{1.3}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} e^x = e^x \tag{1.4}$$

Don't forget the calc 1 derivatives and integrals of polynomials and basic trig functions:

$$\frac{\mathrm{d}}{\mathrm{d}x} x^n = nx^{n-1}; \int x^n \mathrm{d}x = \frac{x^{n+1}}{n+1} + C \tag{1.5}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \sin(x) = \cos(x); \int \sin(x) = -\cos(x) + C \tag{1.6}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \cos(x) = -\sin(x); \int \cos(x) = \sin(x) + C \tag{1.7}$$

Basic trig identities are assumed known and memorized as prerequisite material:

$$\tan(x) = \frac{\sin(x)}{\cos(x)}, \cot(x) = \frac{\cos(x)}{\sin(x)}, \csc(x) = \frac{1}{\sin(x)}, \sec(x) = \frac{1}{\cos(x)} \tag{1.8}$$

The most useful trig identity:

$$\sin^2(x) + \cos^2(x) = 1 \tag{1.9}$$

Equations (1.6) and (1.7) remind us of the similar (but **different!**) formulas for hyperbolic trig functions

$$\frac{\mathrm{d}}{\mathrm{d}x} \sinh(x) = \cosh(x) \tag{1.10}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \cosh(x) = \sinh(x) \tag{1.11}$$

You should have the following two **memorized** for convenience:

$$\int \frac{1}{\sqrt{a^2 - x^2}} \mathrm{d}x = \arcsin\left(\frac{x}{a}\right) + C \tag{1.12}$$

$$\int \frac{1}{a^2 + x^2} \mathrm{d}x = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C \tag{1.13}$$

If you remember that changing the the sign of the variable inside the integrand yields hyperbolics, then (1.12) and (1.13) give you the following "for free"

$$\int \frac{1}{\sqrt{a^2+x^2}}\mathrm{d}x = \operatorname{arcsinh}\left(\frac{x}{a}\right) + C \tag{1.14}$$

$$\int \frac{1}{a^2-x^2}\mathrm{d}x = \frac{1}{a}\operatorname{arctanh}\left(\frac{x}{a}\right) + C \tag{1.15}$$

FTC applied to (1.12), (1.13), (1.14), and (1.15) give us (taking $a = 1$ in each case) the following "for free":

$$\frac{\mathrm{d}}{\mathrm{d}x}\arcsin(x) = \frac{1}{\sqrt{1-x^2}} \tag{1.16}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\arctan(x) = \frac{1}{1+x^2} \tag{1.17}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\operatorname{arcsinh}(x) = \frac{1}{\sqrt{1+x^2}} \tag{1.18}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\operatorname{arctanh}(x) = \frac{1}{1-x^2} \tag{1.19}$$