§9.10

#1 Well-Known: $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

Thus,

$$e^{2x} = \sum_{k=0}^{\infty} \frac{2^k x^k}{k!}$$

#2 Well-known: $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$

Thus,

$$e^{-4x} = \sum_{k=0}^{\infty} \frac{(-1)^k 4^k x^k}{k!}$$

#8 Well-known: $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$,

BUT that is centered at $c=0$, not $c=1$ like required. Hence we proceed by def of Taylor Series:

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

Let $f(x) = e^x$, then for $c=1$,

$f^{(0)}(x) = e^x \rightarrow f^{(0)}(1) = e$

$f^{(1)}(x) = e^x \rightarrow f^{(1)}(1) = e$

$\vdots \qquad\qquad \vdots$

$f^{(k)}(x) = e^x \rightarrow f^{(k)}(1) = e$

Therefore,

$$e^x = \sum_{k=0}^{\infty} \frac{e(x-1)^k}{k!}$$

Problem A

$$f(x) = \sin(x)\cos(x), \quad c = \frac{\pi}{2}$$

$$f^{(0)}(x) = \sin(x)\cos(x)$$

$$f^{(1)}(x) = \frac{d}{dx}\sin(x)\cos(x) = \cos^2(x) - \sin^2(x)$$

$$f^{(2)}(x) = 2\cos(x)(-\sin(x)) - 2\sin(x)\cos(x)$$

So,

$$f^{(0)}(\pi/2) = \sin(\pi/2)\cos(\pi/2) = 0$$

$$f^{(1)}(\pi/2) = \cos^2(\pi/2) - \sin^2(\pi/2) = -1$$

$$f^{(2)}(\pi/2) = 2\cos(\pi/2)(-\sin(\pi/2)) - 2\sin(\pi/2)\cos(\pi/2) = 0 - 0 = 0$$

Therefore

$$\sin(x)\cos(x) = \underbrace{0 + \frac{(-1)}{1!}(x - \pi/2) + 0}_{\text{first three terms}} + \ldots$$