§9.8 #6 $\sum_{n=0}^{\infty} (3x)^n$ ~ geometric with $r = 3x$

$\Downarrow$

converges whenever

$-1 < r < 1,$

i.e.

$-1 < 3x < 1$

$\Downarrow$

$-\frac{1}{3} < x < \frac{1}{3}$

$\Rightarrow \begin{cases} IoC = (-\frac{1}{3}, \frac{1}{3}) \\ RoC = 1/3 \end{cases}$

§9.8 #7 $\sum_{n=1}^{\infty} \frac{(4x)^n}{n^2}$

Ratio test: $\lim_{n\to\infty} \left| \frac{(4x)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(4x)^n} \right|$

$= \lim_{n\to\infty} \left| 4x \cdot \underbrace{\frac{n^2}{(n+1)^2}}_{\downarrow 1} \right|$

$= |4x|$

Convergence: $|4x| < 1 \Rightarrow -1 < 4x < 1$

$-1/4 < x < 1/4$

$\Rightarrow RoC = 1/4$

§9.8 #9 $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$

Ratio test: $\lim_{n\to\infty} \left| \frac{x^{2(n+1)}}{(2(n+1))!} \cdot \frac{(2n)!}{x^{2n}} \right|$

$= \lim_{n\to\infty} \left| \frac{x^2}{(2n+2)(2n+1)} \right| = 0$

$\Rightarrow$ converges for all x

$\Rightarrow$ RoC $= \infty$

§9.8 #10 $\sum_{n=0}^{\infty} \frac{(2n)!\, x^{2n}}{n!}$

Ratio test: $\lim_{n\to\infty} \left| \frac{(2(n+1))!\, x^{2(n+1)}}{(n+1)!} \cdot \frac{n!}{(2n)!\, x^{2n}} \right|$

$= \lim_{n\to\infty} \left| \underbrace{\frac{(2n+2)}{n+1}}_{\to 2} \underbrace{(2n+1)}_{\to \infty} \cdot x^2 \right|$

$= \infty$

Thus the series diverges for all $x \neq 0$

$\Rightarrow$ RoC $= 0$.

§9.8 #11 $\sum_{n=0}^{\infty} \left(\frac{x}{4}\right)^n \sim$ geometric $\Rightarrow -1 < \frac{x}{4} < 1$ ← endpts diverge for geometric!

$\Rightarrow -4 < x < 4$

$\Rightarrow$ IoC $= (-4, 4)$

RoC $= 4$

§9.8 #13 $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n} \sim$ Ratio test: $\lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{n+1}}{n+1} \cdot \frac{n}{(-1)^n x^n} \right|$

$= \lim_{n\to\infty} \left| \underbrace{\frac{n}{n+1}}_{\to 1} x \right|$

$= |x|$

converge	diverge	inconclusive
$\lvert x\rvert < 1$	$\lvert x\rvert > 1$	$\lvert x\rvert = 1$
$-1 < x < 1$	$x > 1$ or $x < -1$	$x = 1, -1$

Check convergence at $x=1$ ✓

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sim \text{converges by alternating series test}$$

Check convergence at $x=-1$

$$\sum_{n=1}^{\infty} \frac{(-1)^n(-1)^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n} \sim \text{diverges} \sim \text{harmonic series}$$

Therefore

$$\begin{cases} IoC = [-1,1) \\ RoC = 1 \end{cases}$$

§9.8 #19

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{6^n} \sim \text{Ratio test:}$$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+2} x^{n+1}}{6^{n+1}} \cdot \frac{6^n}{(-1)^n x^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x}{6} \right|$$

$$= \left| \frac{x}{6} \right|$$

converges	diverges	inconclusive
$\lvert x/6 \rvert < 1$	$\lvert \frac{x}{6} \rvert > 1$	$\lvert \frac{x}{6} \rvert = 1$
$\downarrow$		$\downarrow$
$-6 < x < 6$		$x = \pm 6$

Check endpt $x=6$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 6^n}{6^n} = \sum_{n=1} (-1)^{n+1} \sim \text{diverge (oscillating partial product)}$$

Check endpt $x=-6$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-6)^n}{6^n} = \sum_{n=1}^{\infty} (-1) \sim \text{diverges (limit of summand is not zero)}$$

$$\Rightarrow IoC = (-6,6)$$

$$RoC = 6$$

§9.8 #20 $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n n!\,(x-5)^n}{3^n}$ ~ Ratio test

$$\lim_{n\to\infty} \left| \frac{(n+1)!\,(x-5)^{n+1}}{3^{n+1}} \cdot \frac{3^n}{n!\,(x-5)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(n+1)(x-5)}{3} \right| = \infty$$

$\Rightarrow$ Series diverges except at center $x=5$

$$\Rightarrow \begin{cases} IoC = [5,5] \\ RoC = 0 \end{cases}$$

§9.8 #25 $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^{n-1}}{3^{n-1}}$ ~ root test:

$$\lim_{n\to\infty} \sqrt[n-1]{\left|\frac{(x-3)^{n-1}}{3^{n-1}}\right|}$$

$$= \lim_{n\to\infty} \left|\frac{x-3}{3}\right|$$

$$= \left|\frac{x-3}{3}\right|$$

converges: $\left|\frac{x-3}{3}\right| < 1$

$\Downarrow$

$-1 < \frac{x-3}{3} < 1$

$-3 < x-3 < 3$

$0 < x < 6$

diverges: $\left|\frac{x-3}{3}\right| > 1$

inconclusive: $\left|\frac{x-3}{3}\right| = 1$

$\downarrow$

$\frac{x-3}{3} = \pm 1$

$\swarrow$ OR $\searrow$

$\frac{x-3}{3} = 1$ $\frac{x-3}{3} = -1$

$x=6$ $x=0$

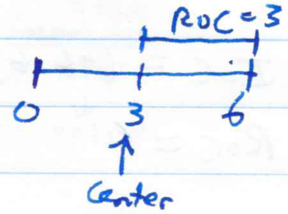
Check endpt $x=0$

$$\sum_{n=1}^{\infty} \frac{(0-3)^{n-1}}{3^{n-1}} = \sum_{n=1}^{\infty} (-1)^{n-1} \sim \text{diverges (oscillating partial sum)}$$

Check endpt $x=6$

$$\sum_{n=1}^{\infty} \frac{(6-3)^{n-1}}{3^{n-1}} = \sum_{n=1}^{\infty} 1 \sim \text{diverges to } \infty$$

$$\Rightarrow \begin{cases} IoC = (0,6) \\ RoC = 3 \end{cases}$$



X

§9.8 #27 $\sum_{n=1}^{\infty} \frac{n}{n+1}(-2x)^{n-1}$ // Ratio test: $\lim_{n\to\infty}\left|\frac{n+1}{n+2}(-2x)^{n}\cdot\frac{n+1}{n(-2x)^{n-1}}\right|$

$= \lim_{n\to\infty}\left|\underbrace{\frac{(n+1)}{(n+2)}}_{\downarrow 1}\underbrace{\frac{(n+1)}{n}}_{\downarrow 1}\cdot\left(\frac{2x}{\ }\right)\right|$

$= |2x|$

conv	div	inconclu
$\lvert 2x\rvert<1$	$\lvert 2x\rvert>1$	$\lvert 2x\rvert=1$
$\downarrow$		$\downarrow$
$-1<2x<1$		$x=\pm\frac{1}{2}$
$\downarrow$		
$-\frac{1}{2}<x<\frac{1}{2}$		

Check endpt $x=-\frac{1}{2}$

$\sum_{n=1}^{\infty}\frac{n}{n+1}(1)^{n-1}$ ~ diverges b/c $\lim_{n\to\infty}\frac{n}{n+1}\neq 0$

Check endpt $x=\frac{1}{2}$

$\sum_{n=1}^{\infty}\frac{n}{n+1}(-1)^{n-1}$ ~ diverges b/c $\lim_{n\to\infty}\frac{n(-1)^{n-1}}{n+1}\neq 0$

$\Rightarrow \begin{cases} IoC = \left(-\frac{1}{2},\frac{1}{2}\right) \\ RoC = \frac{1}{2}\end{cases}$

§9.9 #6 $f(x)=\frac{2}{6-x}$; $c=-2$ $\longrightarrow$ Think: $\sum(x-2)^k=\frac{1}{1-(x-2)}$

$=\frac{1}{1-x+2}$

~~Rewrite~~

$\Rightarrow f(x)=\frac{2}{6-x+2-2}$

$=\frac{2}{4-(x-2)}=\frac{2}{4-(x-2)}\frac{(1/4)}{(1/4)}=\frac{1/2}{1-\left(\frac{x-2}{4}\right)}$

Therefore

$f(x)=\frac{1}{2}\sum_{k=0}^{\infty}\left(\frac{x-2}{4}\right)^k$

Geometric $\Rightarrow$ IoC is given by ~~#2~~

endpts diverge

$-1<\frac{x-2}{4}<1 \to -4<x-2<4$

$\to -2<x<6$

$\Rightarrow IoC=(-2,6)$

X

§9.9 #8 $\frac{1}{1-5x}$; $c=0$

↑ already in ok form

$\Rightarrow \frac{1}{1-5x} = \sum_{k=0}^{\infty} (5x)^k$

IoC: $-1 < 5x < 1 \rightarrow -\frac{1}{5} < x < \frac{1}{5}$

$\rightarrow \text{IoC} = (-1/5, 1/5)$

§9.9 #10 $\frac{3}{2x-1}$; $c=2 \rightsquigarrow$ Think $\sum (x-2)^k = \frac{1}{1-(x-2)} = \frac{1}{1-x+2}$

$$\frac{3}{2x-1} = \frac{3/2}{x-1/2} = \frac{3/2}{x-1/2-2+2} = \frac{3/2}{}$$

$$= \frac{3/2}{3/2+(x-2)}$$

$$= \frac{3/2}{3/2-(2-x)}\left(\frac{2/3}{2/3}\right)$$

$$= \frac{1}{1-(2-x)(2/3)}$$

$$= \sum_{k=0}^{\infty}\left(\frac{2(2-x)}{3}\right)^k$$

IoC $\rightsquigarrow -1 < \frac{2(2-x)}{3} < 1 \longrightarrow -3 < 4x-2 < 3$

$\longrightarrow -1 < 4x < 5$

$\longrightarrow -1/4 < x < 5/4$

$\Rightarrow \text{IoC} = (-1/4, 5/4)$

§9.9 #19 Find series for $\frac{-1}{(1+x)^2}$.

(geometric) IoC $= (-1,1)$

Start with $\frac{1}{1+x} = \frac{1}{1-(-x)} = \sum_{k=0}^{\infty} (-x)^k$

Then $\Downarrow$ d/dx both sides

$$-\frac{1}{(1+x)^2} = \frac{d}{dx}\sum (-x)^k$$

$$= -\sum_{k=0}^{\infty} k(-x)^{k-1}$$

$\Rightarrow \text{IoC} = (-1,1)$

§9.9 #21 Find series for $\log(x+1)$

Start with $\dfrac{1}{x+1} = \dfrac{1}{1-(-x)} = \displaystyle\sum_{k=0}^{\infty}(-x)^k$

Then

$$\Downarrow \int - \,dx \text{ both sides}$$

$$\ln(x+1) = \int \sum_{k=0}^{\infty}(-x)^k$$

$$= \sum_{k=0}^{\infty} \int (-x)^k \, dx$$

$$= -\sum_{k=0}^{\infty} \frac{(-x)^{k+1}}{(k+1)}$$

§9.9 #36

$$\frac{1}{(1-x)} = \sum_{k=0}^{\infty} x^k \leftarrow \text{IoC} = (-1,1)$$

$$\downarrow d/dx$$

$$\frac{-1}{(1-x)^2} = \sum_{k=0}^{\infty} k x^{k-1}$$

$$\downarrow \text{mult. by } (-x)$$

$$\frac{x}{(1-x)^2} = -\sum_{k=0}^{\infty} k x^k$$