Pg. 86 # 2 (translate to symbols)

Premises: $\left[\begin{array}{l}(\forall x)(Bx \to Sx) \\ (\exists x)(Px \wedge Sx)\end{array}\right.$

conclu: $(\exists x)(Px \wedge Bx)$

Pg. 86 #2 Show that an argument that claims to deduce

$$(\exists x)(Px \wedge Bx)$$

from premises

$$(\forall x)(Bx \to Sx) \quad \text{and}$$

$$(\exists x)(Px \wedge Sx)$$

is not valid.

Soln: We define an interpretation:

$$\mathcal{U} = \{-1, 0, 1\}$$

$$Bx = \text{"} x = -1 \text{"}$$

$$Sx = \text{"} x \neq 1 \text{"}$$

$$Px = \text{"} x = 0 \text{"}$$

Then premises

$$(\forall x)(Bx \to Sx) \sim (\forall x)(x = -1 \to x \neq 1)$$

and

$$(\exists x)(Px \wedge Sx) \text{ — } (\exists x)(x = 0 \wedge x \neq 1)$$

are true while the conclusion

$$(\exists x)(Px \wedge Bx) \text{ — } (\exists x)(x = 0 \wedge x = -1)$$

is false.

pg.86 | #6 (translate)

premises $\begin{cases} (\exists x)(Sx \wedge Hx) \\ (\exists x)(Sx \wedge \neg Bx) \end{cases}$

Conclusion : $(\exists x)(Hx \wedge \neg Bx)$

pg.86 #6 | Show that an argument that claims to deduce

$$(\exists x)(Hx \wedge -Bx)$$

from the premises

$(\exists x)(Sx \wedge Hx)$ and

$(\exists x)(Sx \wedge -Bx)$

is not valid.

Soln: We define an interpretation:

$$\mathcal{U} = \{0, 1\}$$

$Hx =$ "$x = 1$"

$Bx =$ "$x = 1$"

$Sx =$ "$x \geq 0$"

Then the premises

$(\exists x)(Sx \wedge Hx)$ is true (let $x = 1$) and

$(\exists x)(Sx \wedge -Bx)$ is true (let $x = 0$), but

the conclusion $(\exists x)(Hx \wedge -Bx)$ is false.

pg. 86 #7 (translate)

premises $\begin{cases} (\forall x)(Nx \to -Lx) \\ (\exists x)(Lx \wedge Cx) \end{cases}$

conclusion = $(\exists x)(Cx \wedge -Nx)$

pg. 86 #7

From the premises

$$(\forall x)(Nx \to -Lx) \quad \text{and}$$

$$(\exists x)(Lx \wedge Cx)$$

prove

$$(\exists x)(Cx \wedge -Nx).$$

Soln:

$\{1\}$	(1) $(\forall x)(Nx \to -Lx)$	Premise
$\{2\}$	(2) $(\exists x)(Lx \wedge Cx)$	Premise
$\{2\}$	(3) $L\alpha \wedge C\alpha$	2 ES
$\{1\}$	(4) $N\alpha \to -L\alpha$	1 US
$\{1\}$	(5) $-(-L\alpha) \to -N\alpha$	4 Contraposition
$\{2\}$	(6) $L\alpha$	3 Simplification
$\{2\}$	(7) $-(-L\alpha)$	6 Double Negative
$\{1,2\}$	(8) $-N\alpha$	5 7 Detachment
$\{2\}$	(9) $C\alpha$	3 Comm. Law of $\wedge$ and Simplification
$\{1,2\}$	(10) $C\alpha \wedge -N\alpha$	8 9 Adjunction
$\{1,2\}$	(11) $(\exists x)(Cx \wedge -Nx)$	10 EG

86 #8 (translate)

premises: $\begin{cases} (\forall x)(Px \to Rx) \\ (\exists x)(Tx \land -Rx) \end{cases}$

conclusion: $(\exists x)(Tx \land -Px)$

pg-86 #8 From the premises

$(\forall x)(Px \to Rx)$ and

$(\exists x)(Tx \land -Rx)$

prove

$(\exists x)(Tx \land -Px)$.

Solution:

$\{1\}$	(1)	$(\forall x)(Px \to Rx)$	Premise
$\{2\}$	(2)	$(\exists x)(Tx \land -Rx)$	Premise
$\{2\}$	(3)	$T\alpha \land -R\alpha$	2 ES
$\{1\}$	(4)	$P\alpha \to R\alpha$	1 US
$\{1\}$	(5)	$-R\alpha \to -P\alpha$	4 Contraposition
$\{2\}$	(6)	$-R\alpha$	3 Comm. law of $\land$ and Simplification
$\{1,2\}$	(7)	$-P\alpha$	5 6 Detachment
$\{2\}$	(8)	$T\alpha$	3 Simplification
$\{1,2\}$	(9)	$T\alpha \land -P\alpha$	7 8 Adjunction
$\{1,2\}$	(10)	$(\exists x)(Tx \land -Px)$	9 EG