HW 12

① Ack was defined by

$$Ack(x,y) = \begin{cases} y+1 \ ; & x=0 \\ Ack(x-1,1) \ ; & y=0 \\ Ack(x-1, Ack(x,y-1)) \ ; & \text{otherwise} \end{cases}$$

Therefore

$$Ack(1,2) \overset{x=1,y=2}{=} Ack(0, \underbrace{Ack(1,1)}_{x=1,y=1})$$

$$= Ack(0, \underbrace{Ack(1,0)}_{x=1,y=0})$$

$$= Ack(0,1)$$

$$\overset{x=0,y=1}{=} 2$$

$$= Ack(0,2)$$

$$=$$

$$Ack(1,2) \overset{x=1,y=2}{=} Ack(0, Ack(1,1))$$

$$= Ack(0, Ack(0, Ack(1,0)))$$

$$= Ack(0, Ack(0, Ack(0,1)))$$

$$= Ack(0, Ack(0,2))$$

$$= Ack(0,3)$$

$$= 4$$

(2) Definition:

$$\boxed{x(n+1) = (n+1)\,x(n), \quad x(0) = 3}$$

Compute

$\boxed{n=0} \longrightarrow x(1) = 1x(0) = 1(3) = 3 = 3(1!)$

$\boxed{n=1} \longrightarrow x(2) = 2x(1) = 2(3) = 3(2!)$

$\boxed{n=2} \longrightarrow x(3) = 3x(2) = 2\cdot 3\cdot 3 = 3(3!)$

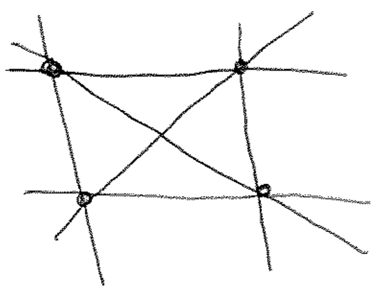
$\boxed{n=3} \longrightarrow x(4) = 4x(3) = 4\cdot 3\cdot 2\cdot 3 = 3(4!)$

$\boxed{n=4} \longrightarrow x(5) = 5x(4) = 5\cdot 4\cdot 3\cdot 2\cdot 3 = 3(5!)$

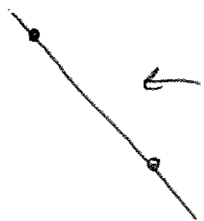
Generally,

$$x(n) = 3(n+1)!$$

(3)

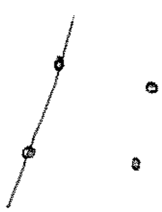


(4)



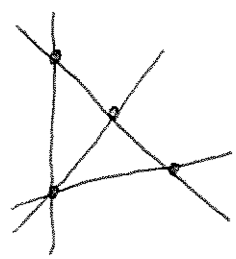
← Axioms 2 and 3 hold, but not Axiom 1

(5)



← Axioms 1 and 3 hold, but not Axiom 2

(6)



← Axioms 1 and 2 hold, but not Axiom 3

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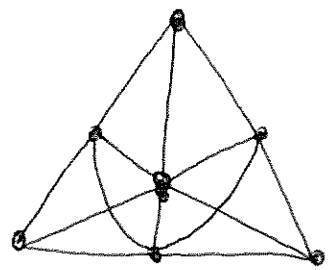

8 Gödel number of $S0 \cdot S0 = S0$

$$2^2\, 3^3\, 5^4\, 7^2\, 11^3\, 13^5\, 17^2\, 19^3$$

Gödel number of $S0 \cdot SS0 = (S0 \cdot S0) + S0$

$$2^2\, 3^3\, 5^4\, 7^2\, 11^2\, 13^3\, 17^5\, 19^6\, 23^2\, 29^3\, 31^4\, 37^2\, 41^3\, 43^7\, 47^8\, 53^2\, 59^3$$

Gödel number of $S0 \cdot SS0 = S0 + S0$

$$2^2\, 3^3\, 5^4\, 7^2\, 11^2\, 13^3\, 17^5\, 19^2\, 23^3\, 29^8\, 31^2\, 37^3$$

9 Super Gödel number of proof

$$2^{2^2 3^3 5^4 7^2 11^3 13^5 17^2 19^3} \cdot 3^{2^2 3^3 5^4 7^2 11^2 13^3 17^5 19^6 23^2 29^3 31^4 37^2 41^3 43^7 47^8 53^2 59^3} \cdot 5^{2^2 3^3 5^4 7^2 11^2 13^3 17^5 19^2 23^3 29^8 31^2 37^3}$$