HW 11

(1) $x=\{a,b,c\}$, $y=\{c,d,e\}$

a) $x \cup y = \{a,b,c,d,e\}$, $x \cap y = \{c\}$

b) $r_1 = \{(1,2),(3,7),(4,9)\}$ ← function

$r_2 = \{(1,1),(1,2),(3,11)\}$ ← not a function

$r_1 \cup r_2 = \{(1,2),(3,7),(4,9),(1,1),(1,2),(3,11)\}$ ← not a function

$r_1 \cap r_2 = \{\ \} = \phi$ ← function ("vacuously")

(2) Theorem A: $(\exists x)(x=\{\phi\})$

{Ax 2}	(1) $\alpha=\{\phi\}$ ~~Theorem A, ES~~	Theorem A, ES
{Ax 2}	(2) $(\exists y)(\forall x)(x\in y \leftrightarrow x=\alpha \vee x=\alpha)$	Axiom 2, US twice
{Ax 2}	(3) $(\exists y)(\forall x)(x\in y \leftrightarrow x=\alpha)$	2 ET (self-or)
{Ax, 2}	(4) $(\exists y)(\forall x)(x\in y \leftrightarrow x=\{\phi\})$	1, 3 Identity

(3) No. The set $\{\dots,-3,-2,-1\}$ is a subset of X, but it has no least element.

(4) a)

(4a)

$\{Ax\ 1\}$ (1) $\partial((f\cdot g)\cdot h) = \partial(f\cdot g)\cdot h + (f\cdot g)(\partial h)$ Ax 1, US twice

$\{Ax\ 1\}$ (2) $\partial(f\cdot g) = (\partial f)\cdot g + f\cdot(\partial g)$ Ax 1, US twice

$\{Ax\ 1\}$ (3) $\partial(f\cdot g) = ((\partial f)\cdot g + f\cdot(\partial g))\cdot h + (f\cdot g)(\partial h)$ 1 2 Identity

(4b) $\{Ax\ 1\}$ (1) $\partial(f\cdot g) = (\partial f)\cdot g + f\cdot(\partial g)$ Ax 1, US twice

$\{Ax\ 2\}$ (2) $\partial((\partial f)\cdot g + f\cdot(\partial g)) = \partial((\partial f)\cdot g) + \partial(f\cdot(\partial g))$ Ax 2, US twice

$\{Ax\ 1,2\}$ (3) $\partial(\partial(f\cdot g)) = \partial((\partial f)\cdot g) + \partial(f\cdot(\partial g))$ 1, 2 Identity

$\{Ax\ 1\}$ (4) $\partial((\partial f)\cdot g) = \partial(\partial f)\cdot g + (\partial f)\cdot(\partial g)$ Axiom 1, US twice

$\{Ax\ 1\}$ (5) $\partial(f\cdot(\partial g)) = (\partial f)\cdot(\partial g) + f\cdot\partial(\partial g)$ Axiom 1, US twice

$\{Ax\ 1,2\}$ (6) $\partial(\partial(f\cdot g)) = (\partial((\partial f)\cdot g) + \partial(f\cdot(\partial g)))$ 3,4,5 Identity

$+ ((\partial f)\cdot(\partial g) + f\cdot(\partial(\partial g)))$