Homework 3 - MATH 2200 Spring 2017

1.) Construct a counterexample to show that the following rule of inference is invalid: from $\neg P$ and $P \to Q$, we may derive $\neg Q$.

Solution: We must pick truth-values for the sentences P and Q that make $\neg P$ and $P \to Q$ true while making $\neg Q$ false. We take Q to be true and P to be false. This means $\neg P$ is true and $P \to Q$ is true (since P is false), while $\neg Q$ is false. This constitutes a counterexample for this being a valid rule of inference.

2.) There is an error in the following deduction. Find it and explain why it is an error. Recall that the **Law of Contraposition** is the following tautological implication: $(P \to Q) \leftrightarrow (\neg Q \to \neg P)$ and the **Law of Detachment** says the following is a tautological implication: $[P \wedge (P \to Q)] \to Q$.

$\{1\}$	$(1) P \to Q$	Premise
$\{2\}$	$(2) Q \to R$	Premise
$\{3\}$	$(3) \neg R$	Premise
$\{2\}$	$(4) \neg R \to \neg Q$	2 T (Law of Contraposition)
$\{2,3\}$	$(5) \neg Q$	3,4 T (Law of Detachment)
$\{1,2,3\}$	$(6) P$	1, 5 T (Law of Detachment)

Solution: The error occurs in line (6), because lines (1) and (5) cannot be used with the Law of Detachment to conclude P.

note: Law of Detachment COULD be used if the law of contraposition was applied to line (1) to get $\neg Q \to \neg P$, but the effect would be to conclude $\neg P$, not P

3.) Is the following formal argument valid? Recall that the **Law of Hypothetical Syllogism** says that $[(P \to Q) \wedge (Q \to R)] \to (P \to R)$ is a tautological implication. Let P denote the sentence "$1+1=3$", let Q denote "$1=2$", and let R denote "$0=1$".

$\{1\}$	$(1) P \to Q$	Premise
$\{2\}$	$(2) Q \to R$	Premise
$\{3\}$	$(3) P$	Premise
$\{1,2\}$	$(4) P \to R$	1,2 T (Law of Hypothetical Syllogism)
$\{1,2,3\}$	$(5) R$	3,4 T (Law of Detachment)

Solution: Yes it is valid. It may "feel" invalid because each of the premises P, Q, and R are false sentences and the conclusion made in line (5) is also false. However there is no problem in this argument; notice that all of our premises are conditional arguments. For example, in line (2) the sentence $Q \to R$ stands in for the sentence "if $1 = 2$, then $0 = 1$", which typical experience in algebra suggests is true ("subtract 1 from each side of the equation"). The validity of this argument and the fact that its conclusion is absurd is itself evidence that the premises are not "good" premises.

4.) Construct a formal deduction for the given argument. Do so by applying the **Law of Absurdity**, which says that $[(P \to Q) \wedge \neg Q] \to \neg P$:

"If the Thomae function is differentiable, then the Thomae function is continuous. The Thomae function is not continuous. Therefore the Thomae function is not differentiable."

Solution: Let P stand for the sentence "The Thomae function is differentiable." and let Q stand for the sentence "The Thomae function is continuous". Construct the formal deduction as described:

$\{1\}$	$(1) P \to Q$	Premise
$\{2\}$	$(2) \neg Q$	Premise
$\{1,2\}$	$(3) \neg P$	1,2 T (Law of Absurdity)

5.) Construct a formal deduction for the given argument. Do so by applying the **Law of Detachment**:

"If the function f is entire, then the contour integral of f over the circle is zero. The function f is entire. Therefore the contour integral of f over the circle is zero."

Solution: Let P denote the sentence "The function f is entire." and let Q denote the sentence "The contour integral of f over the circle is zero." Construct the formal deduction as described:

$\{1\}$	$(1) P \to Q$	Premise
$\{2\}$	$(2) P \to Q$	Premise
$\{1,2\}$	$(3) Q$	1,2 T (Law of Detachment)