MATH 1540 - EXAM 2 - FALL 2017
SOLUTION

Wed/Thu 4/5 October 2017
Instructor: Tom Cuchta

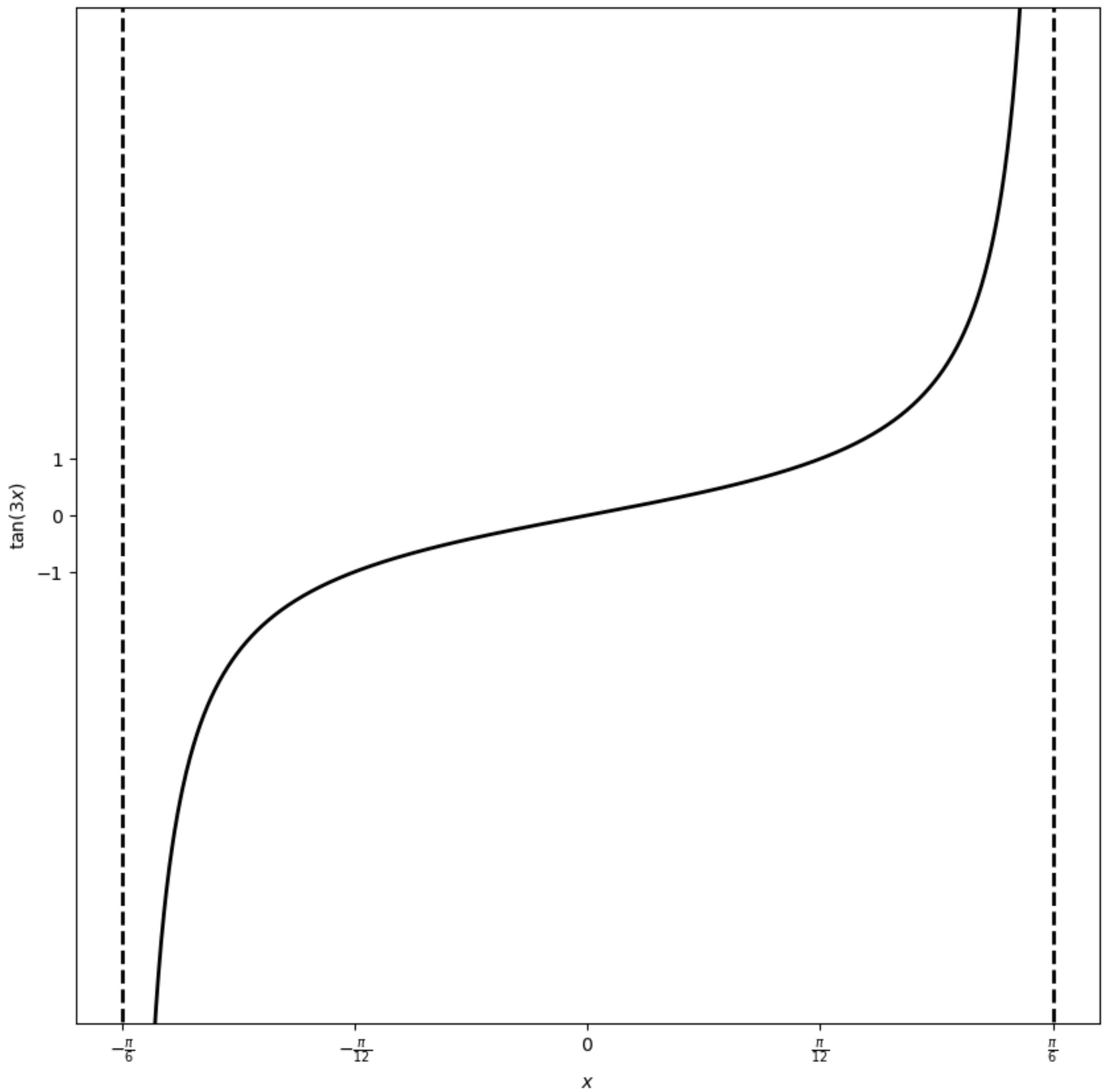
Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (16 points) Graph the function. Include all relevant labels on the x-axis and y-axis for full credit.

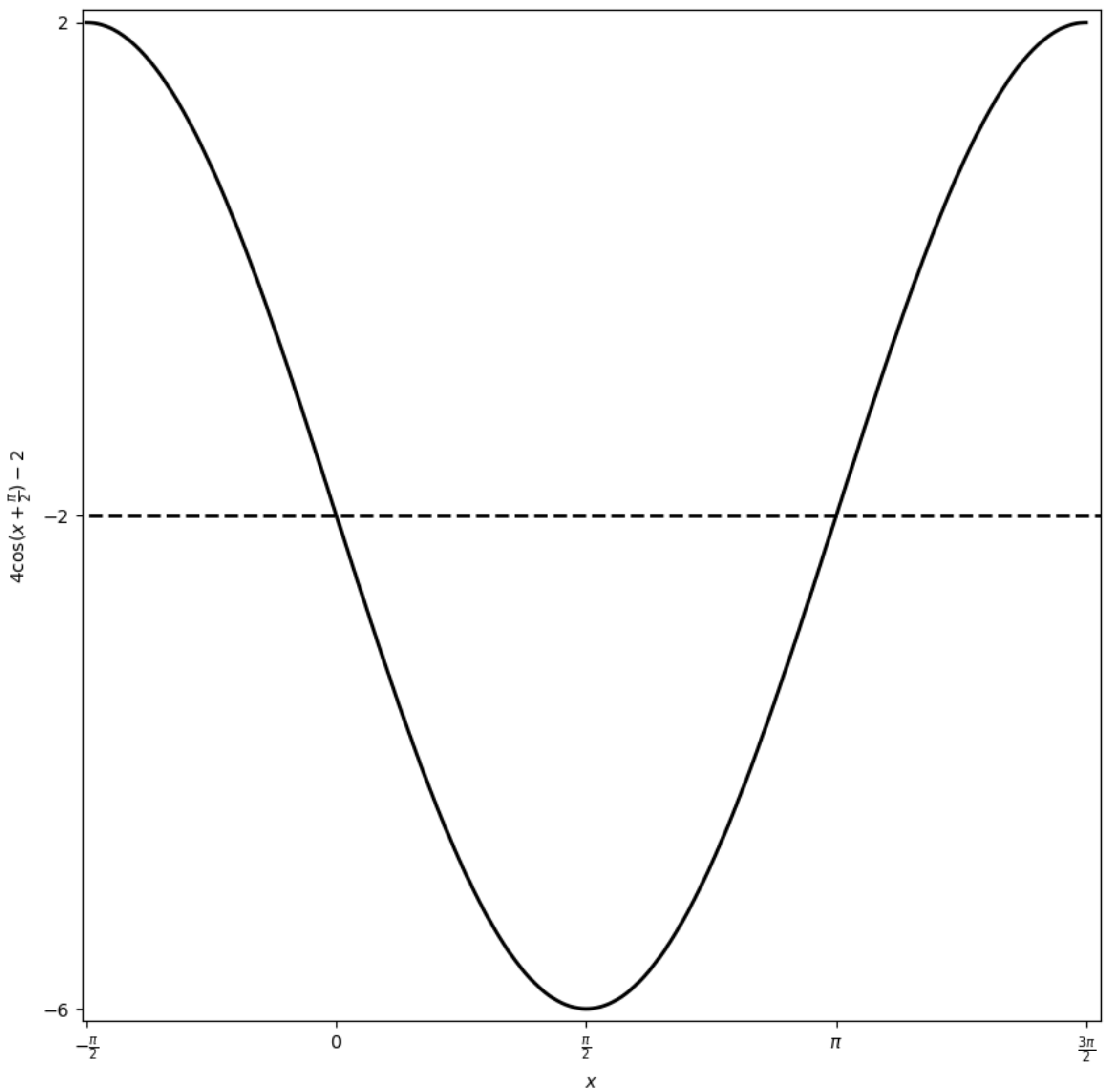
   (a) (8 points) Graph $y = \tan(3x)$.

   Solution: Start with the standard anchor points for tangent: $-\frac{\pi}{2}, -\frac{\pi}{4}, 0, \frac{\pi}{4}, \frac{\pi}{2}$. The "$3x$" informs us to divide each anchor point by 3 (horizontal compression). Therefore the anchor points we use will be $-\frac{\pi}{6}, -\frac{\pi}{12}, 0, \frac{\pi}{12}, \frac{\pi}{6}$. We may now plot:



(b) (8 points) Graph $y = 4\cos\left(x + \frac{\pi}{2}\right) - 2$.

Solution: Start with the standard anchor points for cosine: $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$. The "$x + \frac{\pi}{2}$" that appears in the argument to the cosine function tells us to subtract $\frac{\pi}{2}$ from each anchor point (horizontal shift), yielding $-\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$. The "4" in the front tells us to make the cosine go from -4 to 4 (vertical stretch), but the "-2" tells us to shift these values down by 2 (vertical shift) meaning that the transformed cosine will go between -6 and 2 with a zero line at -2. We may now plot:



2. (21 points) Do each part.

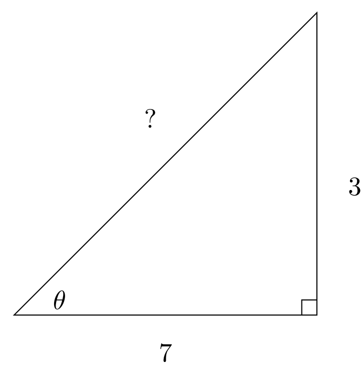
(a) (6 points) Find the exact value of $\cos^{-1}(-1)$.

Solution: Recall that the range of the $\cos^{-1}$ function is $[0, \pi]$, so outputs of this function always lie in quadrant I or quadrant II. Calculating $\cos^{-1}(-1)$ is answering the question "what angle in quadrants I and II has a cosine equal to -1?". Therefore we compute

$$\cos^{-1}(-1) = -\pi.$$

(b) (7 points) Find the exact value of $\sin\left(\tan^{-1}\left(-\frac{3}{7}\right)\right)$.

Solution: Let $\theta = \tan^{-1}\left(-\frac{3}{7}\right)$. From this we may conclude that θ lies in quadrant I or quadrant IV (due to the range of $\tan^{-1}$). We see that $\tan(\theta) = -\frac{3}{7}$, and so since the tangent of θ is negative, we conclude that θ must lie in quadrant II or quadrant IV. Therefore θ lies in quadrant IV (and hence, the sine of θ is negative). Now draw a triangle that agrees with $\tan(\theta) = -\frac{3}{7}$:



To find the missing side labelled "?", we use the Pythagorean theorem:

$$7^2 + 3^2 = ?^2,$$

hence

$$49 + 9 = ?^2,$$
$$58 = ?^2,$$

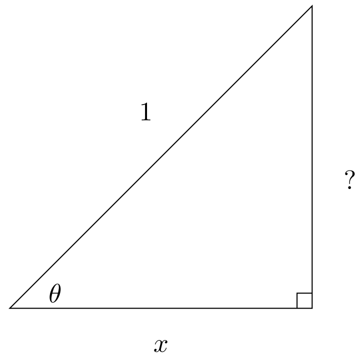
and so

$$? = \sqrt{58}.$$

Therefore we may compute

$$\sin\left(\tan^{-1}\left(-\frac{3}{7}\right)\right) = \sin(\theta) = -\frac{3}{\sqrt{58}}.$$

(c) (8 points) If $x > 0$, find the exact value of $\sin(\cos^{-1}(x))$.
Solution: Let $\theta = \cos^{-1}(x)$, then θ must lie in quadrants I and II. We see that $\cos(\theta) = x$, and (since $x > 0$)) θ must lie in quadrants I or IV. Therefore θ lies in quadrant I. Consequently, $\sin(\theta)$ is positive. Now draw a triangle that agrees with $\cos(\theta) = x$:



We must now find ?. Use the Pythagorean theorem to write

$$x^2 + ?^2 = 1^2.$$

Solving for ? yields

$$? = \sqrt{1 - x^2}.$$

From this we may compute

$$\sin(\cos^{-1}(x)) = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}.$$

3. (16 points) Simplify the first expression and write it in terms of the second expression.
(a) (8 points) $\sin(-x)\csc(x)\sec(x)$; $\cos(x)$
Solution: Recall the "odd" property of sine that says $\sin(-x) = -\sin(x)$. Recall that $\csc(x) = \frac{1}{\sin(x)}$ and $\sec(x) = \frac{1}{\cos(x)}$. We now rewrite this expression and compute

$$\begin{aligned} \sin(-x)\csc(x)\sec(x) &= -\sin(x)\left(\frac{1}{\sin(x)}\right)\left(\frac{1}{\cos(x)}\right) \\ &= -\frac{\sin(x)}{\sin(x)\cos(x)} \\ &= -\frac{1}{\cos(x)}. \end{aligned}$$

We stop there because it is in the desired form.
(b) (8 points) $\frac{\cot(t)+\tan(t)}{\sec(-t)}$; $\sin(t)$
Solution: Recall that $\cot(t) = \frac{\cos(t)}{\sin(t)}$ and $\tan(t) = \frac{\sin(t)}{\cos(t)}$. Recall that $\sec(-t) = \frac{1}{\cos(-t)}$ and the "even" property of cosine that says $\cos(-x) = \cos(x)$. Also recall the Pythagorean identity $\cos^2(x) + \sin^2(x) + 1$. Now compute

$$\begin{aligned} \frac{\cot(t)+\tan(t)}{\sec(-t)} &= \frac{\frac{\cos(t)}{\sin(t)} + \frac{\sin(t)}{\cos(t)}}{\frac{1}{\cos(-t)}} \\ &= \frac{\cos(t)}{1}\left[\frac{\cos^2(t)+\sin^2(t)}{\sin(t)\cos(t)}\right] \\ &= \frac{\cos(t)}{\sin(t)\cos(t)} \\ &= \frac{1}{\sin(t)}. \end{aligned}$$

4. (18 points) The equation $P = 17\sin(2\pi t) + 95$ models someone's blood pressure, P, where t represents time in seconds.

   (a) (5 points) What is the blood pressure after 28 seconds?
   Solution: We set $t = 28$ and compute

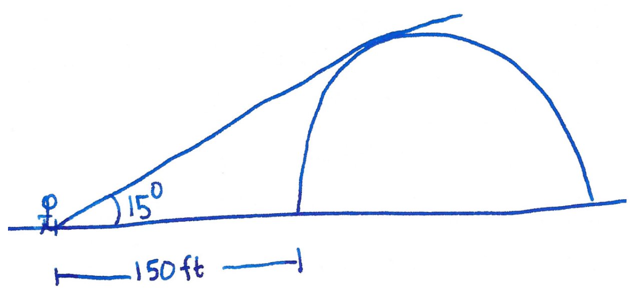
$$P\Big|_{t=28} = 17\sin(2\pi(28)) + 95 = 95.$$

   (b) (6 points) What is the blood pressure after 1 minute?
   Solution: We set $t = 60$ and compute

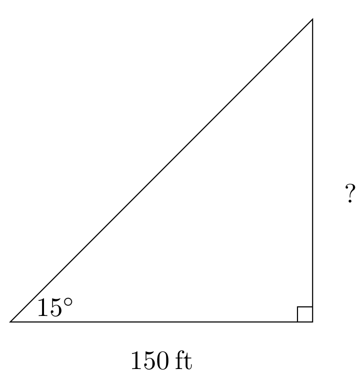
$$P\Big|_{t=60} = 17(2pi(60)) + 95 = 95.$$

   (c) (7 points) What are the maximum and minimum blood pressures?
   Solution: The maximum blood pressure comes from whenever time t obeys $\sin(2pit) = 1$ and the minimum blood pressure comes from whenever time t obeys $\sin(2\pi t) = -1$. Therefore the maximum blood pressure is $17(1) + 95 = 112$ and the minmum blood pressure is $95 - 17 = 78$.

5. (9 points) Suppose that I walk 150 feet away from a the base of a hill and measure the angle of elevation to the top of the hill to be $15°$. How tall is the hill?
Solution: First draw the described situation:



Now draw the relevant right triangle :



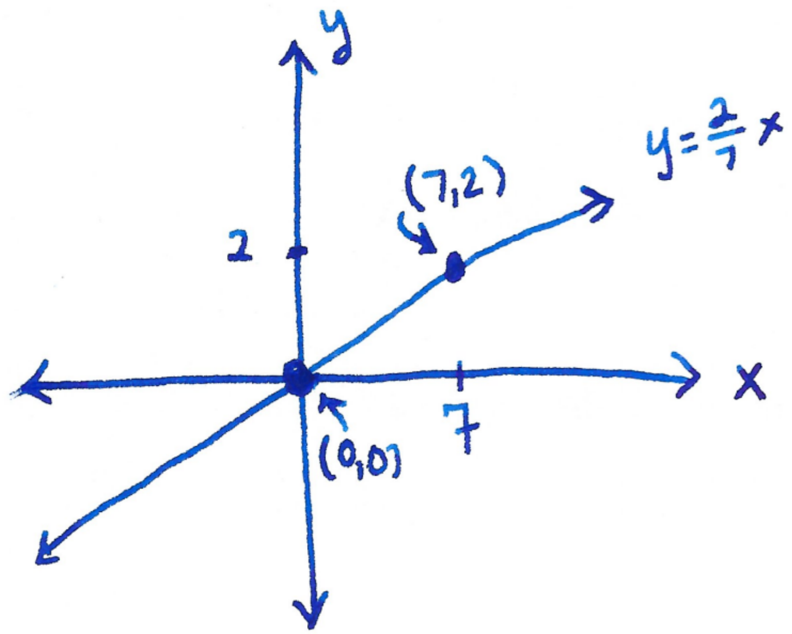
We use the tangent function to relate the angle, the known side, and the unknown side:

$$\tan(15°) = \frac{?}{150}.$$

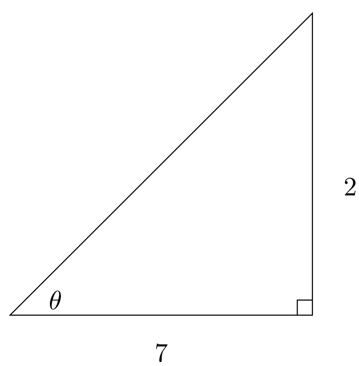
Rearrange this equation to solve for ? and we compute

$$? = 150\tan(15°) \approx 40.19\,\text{ft}.$$

6. (10 points) The line $y = \frac{2}{7}x$ passes through the origin in the xy-plane. What is the angle that the line makes with the x-axis? Express your answer correct to two decimal places.
Solution: First draw the described situation:



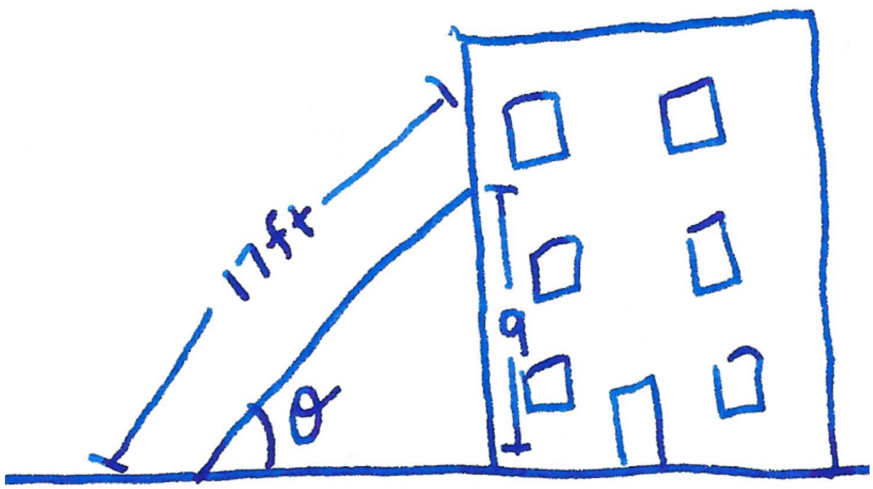
Now draw the relevant right triangle:



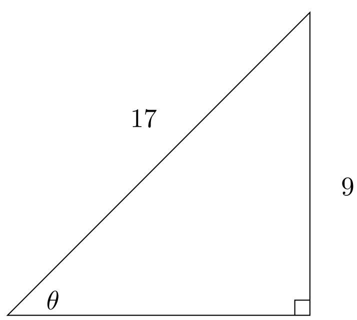
Use the tangent function to write $\tan(\theta) = \frac{2}{7}$. To find the angle θ, apply the inverse tangent function to both sides (recall that $\tan^{-1}(\tan(\theta)) = \theta$) and compute

$$\theta = \tan^{-1}\left(\frac{2}{7}\right) \approx 0.27829\,\text{rad} = 15.95^\circ.$$

7. (10 points) A 17 foot ladder leans against a building so that the top of the ladder is 9 feet from the ground. If safety specifications call for the ladder's angle of elevation to be between 30° and 40°, then does the placement of this ladder obey the specifications?
Solution: First draw this situation:



Now draw the relevant right triangle:



To find θ, first use the sine function to write $\sin(\theta) = \frac{9}{17}$. Now apply the inverse sine function to both sides to compute

$$\theta = \sin^{-1}\left(\frac{9}{17}\right) \approx 31.97°.$$

From this we may conclude that **yes**, the ladder meets the specifications.