MATH 1113 - EXAM 3 SPRING 2017

Name: ______________________________

Friday 21 April 2017
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

$E = z_c\sigma_{\bar{x}}; \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}; \bar{x} - E < \mu < \bar{x} + E$
$E = t_c\dfrac{s}{\sqrt{n}}; \bar{x} - E < \mu < \bar{x} + E$
$E = z_c\sqrt{\dfrac{\hat{p}\hat{q}}{n}}; \hat{p} - E < p < \hat{p} + E$
$\dfrac{(n-1)s^2}{\chi^2_R} < \sigma^2 < \dfrac{(n-1)s^2}{\chi^2_L}$
$\sqrt{\dfrac{(n-1)s^2}{\chi^2_R}} < \sigma < \sqrt{\dfrac{(n-1)s^2}{\chi^2_L}}$

$z = \dfrac{\bar{x} - \mu}{\sigma_{\bar{x}}}; \sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$
$t = \dfrac{\bar{x} - \mu}{s_{\bar{x}}}; s_{\bar{x}} = \dfrac{s}{\sqrt{n}}$
$z = \dfrac{\hat{p} - p}{\sigma_{\hat{p}}}; \sigma_{\hat{p}} = \sqrt{\dfrac{pq}{n}}$
$\chi^2 = \dfrac{(n-1)s^2}{\sigma^2}$
$z = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sigma_{\bar{x}_1,\bar{x}_2}}; \sigma_{\bar{x}_1,\bar{x}_2} = \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$
$t = \dfrac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{s_{\bar{x}_1,\bar{x}_2}}$:   Case I (variances equal): $s_{\bar{x}_1,\bar{x}_2} = \hat{\sigma}\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}; \hat{\sigma} = \sqrt{\dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$;   d.f. $= n_1 + n_2 - 2$ Case II (variances not equal): $s_{\bar{x}_1,\bar{x}_2} = \sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}$ d.f. $=$ smaller of $n_1 - 1$ and $n_2 - 1$

1. (10 points) Construct a 90% confidence interval for the population mean: from a random sample of 48 days in a recent year, U.S. gasoline prices had a mean of \$3.43. Assume the population standard deviation is \$0.19.

   (a) (5 points) Summarize the data in this problem by filling out the table below:

c	$= 0.9$
(Critical value) z_c	= ______
n	= ______
$\bar{x}$	$= 3.43$
______	$= 0.19$
E	= ______

   (b) (5 points) (Fill in the **three** blanks) The confidence interval is:

   ______ < ______ < ______

2. (10 points) Construct a 95% confidence interval for the population proportion: in a survey of 1782 U.S. adults, 659 think that air travel is much more reliable than taking cruises.

   (a) (5 points) Summarize the data in this problem by filling out the table below:

c	$= 0.95$
(Critical value) ______	= ______
n	= ______
$\hat{p}$	= ______
E	= ______

   (b) (5 points) (Fill in the **three** blanks) The confidence interval is:

   ______ < ______ < ______

3. (16 points) A fast food restaurant estimates that the mean sodium content in one of its breakfast sandwiches is no more than 920 milligrams. A random sample of 44 breakfast sandwiches has a mean sodium content of 925 milligrams. Assume the population standard deviation is 18 milligrams. At $\alpha = 0.10$, do you have enough evidence to reject the restaurant's claim?

   (a) (4 points) **Identify the claim** and the hypotheses H_0 and H_a

$$\begin{cases} H_0 : \\ H_a : \end{cases}$$

   (b) (3 points) Summarize the data in this problem by filling out the table below:

α	$= 0.10$
n	$=$ ______
$\bar{x}$	$=$ ______
______	$= 18$

   (c) (2 points) The critical value is $z_0 = 1.28$. What is the rejection region?

   (d) (4 points) Compute the relevant test statistic.

   (e) (2 points) Circle one: Reject H_0 Fail to reject H_0

   (f) (1 point) Write a sentence to interpret the decision made in part (e) for the claim.

4. (16 points) A company claims that the mean battery life of their cell phone is at least 30 hours. You suspect this claim is incorrect and find that a random sample of 18 cell phones has a mean battery life of 28.5 hours and a standard deviation of 1.7 hours. Is there enough evidence to reject the claim at $\alpha = 0.01$? The data in this problem is summarized in the following table:

α	$= 0.01$
n	$= 18$
$\bar{x}$	$= 28.5$
s	$= 1.7$

(a) (4 points) **Identify the claim** and the hypotheses H_0 and H_a

$$\begin{cases} H_0 : \\ H_a : \end{cases}$$

(b) (4 points) What is the critical value for this problem? What is the rejection region?

(c) (4 points) Compute the relevant test statistic.

(d) (2 points) Circle one: Reject H_0 Fail to reject H_0

(e) (2 points) Write a sentence to interpret the decision made in part (d) for the claim.

5. (16 points) A hospital spokesperson claims that the standard deviation of the waiting times experienced by patients in its minor emergency department is no more than 30 seconds. A random sample of 25 waiting times has a standard deviation of 21 seconds. At $\alpha = 0.10$, is there enough evidence to reject the agent's claim?

(a) (4 points) **Identify the claim** and the hypotheses H_0 and H_a

$$\begin{cases} H_0 : \\ H_a : \end{cases}$$

(b) (1 point) Summarize the data in this problem by filling out the table below:

α	$= 0.10$
n	$= 25$
______	$= 21$

(c) (3 points) What is the rejection region?

(d) (4 points) Compute the relevant test statistic.

(e) (2 points) Circle one: Reject H_0 Fail to reject H_0

(f) (2 points) Write a sentence to interpret the decision made in part (e) for the claim.

6. (16 points) To compare braking distances for two types of tires, a safety engineer conducts 35 braking tests for each type. The mean breaking distance for Type A is 42 feet. Assume the population standard deviation for Type A is 4.7 feet. The mean braking distance for Type B is 45 feet. Assume the population standard deviation for Type B is 4.3 feet. At $\alpha = 0.10$, can the engineer support the claim that the mean braking distances for Type A tires are larger than the mean braking distances for Type B tires? The data for this problem is summarized in the following tables:

Type A (μ_1)
$n_1 = 25$
$\bar{x}_1 = 42$
$\sigma_1 = 4.7$

Type B (μ_2)
$n_2 = 25$
$\bar{x}_2 = 45$
$\sigma_2 = 4.3$

(a) (4 points) **Identify the claim** and the hypotheses H_0 and H_a

$$\begin{cases} H_0 : \\ H_a : \end{cases}$$

(b) (3 points) What is the rejection region?

(c) (4 points) Compute the relevant test statistic.

(d) (3 points) Circle one: Reject H_0 Fail to reject H_0

(e) (2 points) Write a sentence to interpret the decision made in part (d) for the claim.

7. (16 points) A personnel director claims that the mean household income is the same in Kauai and Maui. In Kauai, a sample of 18 residents has a mean household income of \$56,900 and a standard deviation of \$12,100. In Maui, a sample of 20 residents has a mean household income of \$57,800 and a standard deviation of \$8000. At $\alpha = 0.10$, can you reject the personnel director's claim? Assume the population variances are not equal. The data for this problem is summarized in the following tables:

Kauai (μ_1)
$n_1 = 18$
$\bar{x}_1 = 56900$
$s_1 = 12100$

Maui (μ_2)
$n_2 = 20$
$\bar{x}_2 = 57800$
$s_2 = 8000$

(a) (4 points) **Identify the claim** and the hypotheses H_0 and H_a

$$\begin{cases} H_0 : \\ H_a : \end{cases}$$

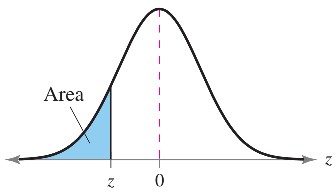
(b) (4 points) What is the rejection region?

(c) (4 points) Compute the relevant test statistic.

(d) (2 points) Circle one: Reject H_0 Fail to reject H_0

(e) (2 points) Write a sentence to interpret the decision made in part (d) for the claim.

Table 4—Standard Normal Distribution



z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
−3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
−3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
−3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
−3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
−3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
−2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
−2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
−2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
−2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047
−2.5	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062
−2.4	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082
−2.3	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107
−2.2	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139
−2.1	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179
−2.0	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228
−1.9	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287
−1.8	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359
−1.7	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446
−1.6	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548
−1.5	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668
−1.4	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808
−1.3	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968
−1.2	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151
−1.1	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357
−1.0	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587
−0.9	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841
−0.8	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119
−0.7	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420
−0.6	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743
−0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085
−0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446
−0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821
−0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207
−0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602
−0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000

Critical Values

Level of Confidence c	z_c
0.80	1.28
0.90	1.645
0.95	1.96
0.99	2.575

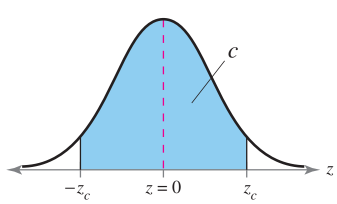
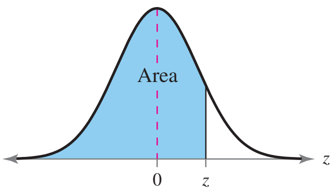


Table 4—Standard Normal Distribution *(continued)*

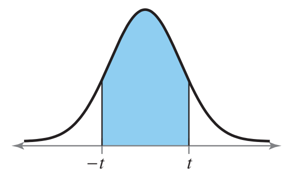


z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

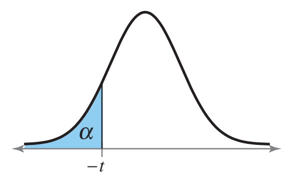
Table 5—*t*-Distribution

d.f.	Level of confidence, *c*	0.80	0.90	0.95	0.98	0.99
	One tail, α	0.10	0.05	0.025	0.01	0.005
	Two tails, α	0.20	0.10	0.05	0.02	0.01
1		3.078	6.314	12.706	31.821	63.657
2		1.886	2.920	4.303	6.965	9.925
3		1.638	2.353	3.182	4.541	5.841
4		1.533	2.132	2.776	3.747	4.604
5		1.476	2.015	2.571	3.365	4.032
6		1.440	1.943	2.447	3.143	3.707
7		1.415	1.895	2.365	2.998	3.499
8		1.397	1.860	2.306	2.896	3.355
9		1.383	1.833	2.262	2.821	3.250
10		1.372	1.812	2.228	2.764	3.169
11		1.363	1.796	2.201	2.718	3.106
12		1.356	1.782	2.179	2.681	3.055
13		1.350	1.771	2.160	2.650	3.012
14		1.345	1.761	2.145	2.624	2.977
15		1.341	1.753	2.131	2.602	2.947
16		1.337	1.746	2.120	2.583	2.921
17		1.333	1.740	2.110	2.567	2.898
18		1.330	1.734	2.101	2.552	2.878
19		1.328	1.729	2.093	2.539	2.861
20		1.325	1.725	2.086	2.528	2.845
21		1.323	1.721	2.080	2.518	2.831
22		1.321	1.717	2.074	2.508	2.819
23		1.319	1.714	2.069	2.500	2.807
24		1.318	1.711	2.064	2.492	2.797
25		1.316	1.708	2.060	2.485	2.787
26		1.315	1.706	2.056	2.479	2.779
27		1.314	1.703	2.052	2.473	2.771
28		1.313	1.701	2.048	2.467	2.763
29		1.311	1.699	2.045	2.462	2.756
30		1.310	1.697	2.042	2.457	2.750
31		1.309	1.696	2.040	2.453	2.744
32		1.309	1.694	2.037	2.449	2.738
33		1.308	1.692	2.035	2.445	2.733
34		1.307	1.691	2.032	2.441	2.728
35		1.306	1.690	2.030	2.438	2.724
36		1.306	1.688	2.028	2.434	2.719
37		1.305	1.687	2.026	2.431	2.715
38		1.304	1.686	2.024	2.429	2.712
39		1.304	1.685	2.023	2.426	2.708
40		1.303	1.684	2.021	2.423	2.704
45		1.301	1.679	2.014	2.412	2.690
50		1.299	1.676	2.009	2.403	2.678
60		1.296	1.671	2.000	2.390	2.660
70		1.294	1.667	1.994	2.381	2.648
80		1.292	1.664	1.990	2.374	2.639
90		1.291	1.662	1.987	2.368	2.632
100		1.290	1.660	1.984	2.364	2.626
500		1.283	1.648	1.965	2.334	2.586
1000		1.282	1.646	1.962	2.330	2.581
∞		1.282	1.645	1.960	2.326	2.576

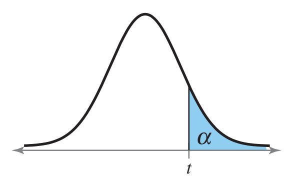
The critical values in Table 5 were generated using Excel 2013.



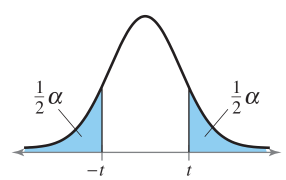
c-confidence interval



Left-tailed test

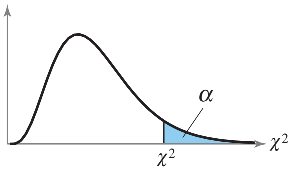


Right-tailed test

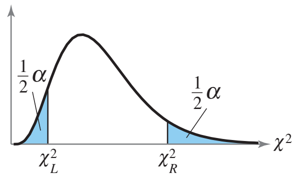


Two-tailed test

Table 6—Chi-Square Distribution



Right tail



Two tails

Degrees of freedom	**α**									
	0.995	**0.99**	**0.975**	**0.95**	**0.90**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**
1	—	—	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
40	20.707	22.164	24.433	26.509	29.051	51.805	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	55.329	85.527	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	73.291	107.565	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.169