

MATH 1113 - EXAM 2 SPRING 2017

SOLUTION

Friday 10 March 2017
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Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (8 points) Consider the following data of grades from some class:

Grade	Number of students
A	5
B	6
C	15
D	4
F	3

What is the probability that a randomly selected student...

- (a) (4 points) has a C?

Solution: The total number of students is

$$5 + 6 + 15 + 4 + 3 = 33.$$

Therefore the probability that a student has a C is $\frac{15}{33} \approx 0.4545$.

- (b) (4 points) has an A or has an F?

Solution: There are 5 students with an A and 3 students with an F, so there are a total of 8 students with an A or an F. Therefore the requested probability is $\frac{8}{33} \approx 0.2424$.

2. (12 points) You roll a 6-sided die. What is the probability of...

- (a) (4 points) rolling a 3 or rolling a 5?

Solution: There are 6 total outcomes of this probability experiment. The event of rolling a 3 or rolling a 5 has size 2, and so the probability of the event is

$$\frac{2}{6} = \frac{1}{3} \approx 0.3333.$$

- (b) (4 points) rolling a 1 or a number greater than 3?

Solution: The outcomes in the event are the numbers 1, 4, 5, and 6 (i.e. 4 total outcomes in the event). Therefore the probability of the event is

$$\frac{4}{6} = \frac{2}{3} \approx 0.6666.$$

- (c) (4 points) rolling an odd number or a number less than 5?

Solution: The outcomes in the event are the numbers 1, 2, 3, 4, 5 (i.e. 5 total outcomes in the event). Therefore the probability of the event is

$$\frac{5}{6} \approx 0.8333.$$

3. (15 points) In a sample of 100 people, 11 of them owned a motorcycle. Two people are selected from this group at random without replacement. Find that probability that...

- (a) (5 points) both of them own a motorcycle.

Solution: This can be understood as

$$\begin{aligned} P(\text{first owns AND second owns}) & \stackrel{\text{mult.rule}}{=} P(\text{first owns})P(\text{second owns} | \text{first owns}) \\ & = \frac{11}{100} \frac{10}{99} \\ & \approx 0.0111 \end{aligned}$$

- (b) (5 points) neither of them own a motorcycle.

Solution: Since we are told that 11 out of the 100 people sampled owned a motorcycle, we see that $100 - 11 = 89$ of the people out of the 100 did not own a motorcycle. The requested probability can be understood as

$$\begin{aligned} & P(\text{first doesn't own AND second doesn't own}) \\ & \stackrel{\text{mult.rule}}{=} P(\text{first doesn't own})P(\text{second doesn't own} \mid \text{first doesn't own}) \\ & = \frac{89}{100} \frac{88}{99} \\ & \approx 0.7911. \end{aligned}$$

- (c) (5 points) at least one of them owns a motorcycle.

Solution: To say that at least one owns a motorcycle is the complementary event to the event in part b where no one owned a motorcycle. So we can simply compute the probability in this event as

$$1 - P(\text{first doesn't own AND second doesn't own}) \approx 1 - 0.7911 = 0.2089.$$

4. (4 points) Construct a probability distribution from the following data describing the number of computers in households in a town:

Computers	0	1	2	3	4
Households	4	8	10	14	3

Solution: First note that the sum of the number of households is

$$4 + 8 + 10 + 14 + 3 = 39.$$

Therefore the probability distribution is

Computers	0	1	2	3	4
Probability	$\frac{4}{39} = 0.1025$	$\frac{8}{39} = 0.2051$	$\frac{10}{39} = 0.2564$	$\frac{14}{39} = 0.3589$	$\frac{3}{39} = 0.0769$

5. (12 points) Consider the following probability distribution:

x	0	1	2	3
$P(x)$	0.11	0.12	0.29	0.48

- (a) (4 points) What is the probability that x is less than 2?

Solution:

$$P(x < 2) = P(x = 0) + P(x = 1) = 0.11 + 0.12 = 0.23.$$

- (b) (4 points) Find the mean of the probability distribution.

Solution: The mean is

$$\text{Mean} = \mu \stackrel{\text{def}}{=} \sum xP(x) = 0(0.11) + 1(0.12) + 2(0.29) + 3(0.48) = 2.14.$$

- (c) (4 points) Find the standard deviation of the probability distribution.

Solution: The standard deviation is

$$\begin{aligned} \text{StdDev} = \sigma & \stackrel{\text{def}}{=} \sqrt{\sum (x - \mu)^2 P(x)} \\ & = \sqrt{(0 - 2.14)^2(0.11) + (1 - 2.14)^2(0.12) + (2 - 2.14)^2(0.29) + (3 - 2.14)^2(0.48)} \\ & = 1.0101. \end{aligned}$$

6. (18 points) (a) (6 points) The probability that you will make a sale on a given telephone call is 0.21. Find the probability that you make your first sale on the sixth call.

Solution: This is a problem involving a geometric random variable with $p = 0.21$ and $q = 1 - p = 0.79$. This variable has distribution $P(x) = (0.21)(0.79)^{x-1}$. Therefore the requested probability is

$$P(x = 6) = 0.21(0.79)^{6-1} = 0.0646.$$

- (b) (6 points) The mean number of oil tankers at a port city is five per day. Find the probability that the number of oil tankers on any given day is at most 2.

Solution: This is a Poisson random variable with $\mu = 5$. This variable has probability distribution $P(x) = \frac{e^{-5}5^x}{x!}$. Therefore the requested probability is

$$P(x \leq 2) = P(x = 0) + P(x = 1) + P(x = 2) = \frac{e^{-5}5^0}{0!} + \frac{e^{-5}5^1}{1!} + \frac{e^{-5}5^2}{2!} = 0.1246$$

- (c) (6 points) Sixty-eight percent of adults would still consider a car brand despite product and safety recalls. You randomly select 4 adults. Find the probability that the number of adults who would consider a car brand despite product and safety recalls is at least 3.

Solution: This is a binomial random variable with $n = 4$, $p = 0.68$, and $q = 1 - p = 0.32$. This variable has probability distribution $P(x) = \frac{4!}{x!(4-x)!}(0.68)^x(0.32)^{4-x}$. Therefore the requested probability is

$$P(x \geq 3) = P(x = 3) + P(x = 4) = \frac{4!}{3!(4-3)!}(0.68)^3(0.32)^{4-3} + \frac{4!}{4!0!}(0.68)^4(0.32)^{4-4} = 0.6162$$

7. (8 points) Consider the standard normal distribution with mean $\mu = 0$ and standard deviation $\sigma = 1$.

- (a) (4 points) Find the area under the standard normal to the left of $z = 1.22$.

Solution:

$$P(z < 1.22) = 0.8888.$$

- (b) (4 points) Find the area under the standard normal between $z = -1.53$ and $z = 0$.

Solution:

$$P(-1.53 < z < 0) = P(z < 0) - P(z < -1.53) = 0.5000 - 0.0630 = 0.4370.$$

8. (11 points) The monthly utility bills in a city are normally distributed with mean \$90.23 and standard deviation of \$10.60. Find the probability that a randomly selected utility bill is between \$85 and \$105.

Solution: We are told that $\mu = 90.23$ and $\sigma = 10.60$. We are asked to find $P(85 < x < 105)$. We convert to z -scores:

$$\frac{85 - 90.23}{10.60} < \frac{x - \mu}{\sigma} < \frac{105 - 90.23}{10.60},$$

or equivalently written,

$$-0.49 < z < 1.39.$$

Therefore the requested probability is

$$\begin{aligned} P(85 < x < 105) &= P(-0.49 < z < 1.39) \\ &= P(z < 1.39) - P(z < -0.49) \\ &= 0.9177 - 0.3121 \\ &= 0.6056. \end{aligned}$$

9. (12 points) The average time spent sleeping (in hours) for a group of medical residents at a hospital can be approximated by a normal distribution with mean 6.1 and standard deviation 1.3. What is the shortest amount of time spent sleeping that would still place a resident in the top 5% of sleeping times?
Solution: We are told that $\mu = 6.1$ and $\sigma = 1.3$. Being in the top 5% of sleeping times means being of percentile P_{95} . First we must find a z -value that corresponds (approximately) to the probability 0.95. Looking at the table, we see that this probability occurs between $z = 1.64$ and $z = 1.65$. Average these z -values to obtain $z = 1.645$. Recall that $z = \frac{x - \mu}{\sigma}$ and rearrange this equation algebraically to get $x = z\sigma + \mu$. Now we plug in the known information to find x (i.e. the shortest amount of time spent sleeping that would place a resident in the top 5%) to be

$$x = 1.645(1.3) + 6.1 = 8.2385.$$

Standard normal table for negative z

z	.09	.08	.07	.06	.05	.04	.03	.02	.01	.00
-3.4	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
-3.3	.0003	.0004	.0004	.0004	.0004	.0004	.0004	.0005	.0005	.0005
-3.2	.0005	.0005	.0005	.0006	.0006	.0006	.0006	.0006	.0007	.0007
-3.1	.0007	.0007	.0008	.0008	.0008	.0008	.0009	.0009	.0009	.0010
-3.0	.0010	.0010	.0011	.0011	.0011	.0012	.0012	.0013	.0013	.0013
-2.9	.0014	.0014	.0015	.0015	.0016	.0016	.0017	.0018	.0018	.0019
-2.8	.0019	.0020	.0021	.0021	.0022	.0023	.0023	.0024	.0025	.0026
-2.7	.0026	.0027	.0028	.0029	.0030	.0031	.0032	.0033	.0034	.0035
-2.6	.0036	.0037	.0038	.0039	.0040	.0041	.0043	.0044	.0045	.0047
-2.5	.0048	.0049	.0051	.0052	.0054	.0055	.0057	.0059	.0060	.0062
-2.4	.0064	.0066	.0068	.0069	.0071	.0073	.0075	.0078	.0080	.0082
-2.3	.0084	.0087	.0089	.0091	.0094	.0096	.0099	.0102	.0104	.0107
-2.2	.0110	.0113	.0116	.0119	.0122	.0125	.0129	.0132	.0136	.0139
-2.1	.0143	.0146	.0150	.0154	.0158	.0162	.0166	.0170	.0174	.0179
-2.0	.0183	.0188	.0192	.0197	.0202	.0207	.0212	.0217	.0222	.0228
-1.9	.0233	.0239	.0244	.0250	.0256	.0262	.0268	.0274	.0281	.0287
-1.8	.0294	.0301	.0307	.0314	.0322	.0329	.0336	.0344	.0351	.0359
-1.7	.0367	.0375	.0384	.0392	.0401	.0409	.0418	.0427	.0436	.0446
-1.6	.0455	.0465	.0475	.0485	.0495	.0505	.0516	.0526	.0537	.0548
-1.5	.0559	.0571	.0582	.0594	.0606	.0618	.0630	.0643	.0655	.0668
-1.4	.0681	.0694	.0708	.0721	.0735	.0749	.0764	.0778	.0793	.0808
-1.3	.0823	.0838	.0853	.0869	.0885	.0901	.0918	.0934	.0951	.0968
-1.2	.0985	.1003	.1020	.1038	.1056	.1075	.1093	.1112	.1131	.1151
-1.1	.1170	.1190	.1210	.1230	.1251	.1271	.1292	.1314	.1335	.1357
-1.0	.1379	.1401	.1423	.1446	.1469	.1492	.1515	.1539	.1562	.1587
-0.9	.1611	.1635	.1660	.1685	.1711	.1736	.1762	.1788	.1814	.1841
-0.8	.1867	.1894	.1922	.1949	.1977	.2005	.2033	.2061	.2090	.2119
-0.7	.2148	.2177	.2206	.2236	.2266	.2296	.2327	.2358	.2389	.2420
-0.6	.2451	.2483	.2514	.2546	.2578	.2611	.2643	.2676	.2709	.2743
-0.5	.2776	.2810	.2843	.2877	.2912	.2946	.2981	.3015	.3050	.3085
-0.4	.3121	.3156	.3192	.3228	.3264	.3300	.3336	.3372	.3409	.3446
-0.3	.3483	.3520	.3557	.3594	.3632	.3669	.3707	.3745	.3783	.3821
-0.2	.3859	.3897	.3936	.3974	.4013	.4052	.4090	.4129	.4168	.4207
-0.1	.4247	.4286	.4325	.4364	.4404	.4443	.4483	.4522	.4562	.4602
-0.0	.4641	.4681	.4721	.4761	.4801	.4840	.4880	.4920	.4960	.5000

Standard normal table for positive z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998