MATH 1113 - EXAM 1 SPRING 2017
SOLUTION

Friday 10 February 2017
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (2 points) Answer the following questions.

   (a) (1 point) Is a population or a sample being described?

   "The cholesterol levels of 20 patients in a hospital with 100 patients."

   Solution: This is a sample – the cholesterol levels of entire population of the hospital is not being described, only a part of it.

   (b) (1 point) Is a parameter or a statistic being described?

   "In a recent year, the average math score on the ACT for all graduates was 21.1."

   Solution: This is a parameter – the average was taken among **all** scores on the ACT.

2. (5 points) Consider the following data:

7	40	13	9	25	8	22	11
0	2	18	2	30	7	35	39
12	15	8	6	5	29	0	16

Construct a frequency distribution for the data using 5 classes.

Solution: Note the range is

$$\text{range} = 40 - 0 = 40,$$

so we construct the frequency table as follows:

Class	Frequency
0–8	10
9–16	6
15–24	2
25–32	3
33–40	3

3. (9 points) Consider the following data representing the time (in seconds) of the winning time in the Kentucky Derby between 2000 and 2010:

   119.9 121.0 121.1 121.1 121.3 121.8 122.1 122.6 122.7 124.0 124.0

   (a) (5 points) Find the median.

   Solution: The median – i.e. the middle value — is 121.8. (We did not have to reorder this data because it was already ordered.)

   (b) (4 points) Find the mode.

   Solution: The mode – i.e. the most often occurring value – is both 121.1 and 124.0.

4. (6 points) Find the weighted mean score of the following scores from a hypothetical student in MATH 1113 at the end of the course:

	Score (out of 100 points)	**Percent of final grade**
Homework	74	16 %
Exam 1	80	17 %
Exam 2	70	17 %
Exam 3	68	17 %
Final Exam	86	33 %

Solution: We compute the weighted mean:

$$\begin{aligned}\text{WeightedMean} &= \frac{74(0.16) + 80(0.17) + 70(0.17) + 68(0.17) + 86(0.33)}{0.16 + 0.17 + 0.17 + 0.17 + 0.33} \\ &= \frac{77.28}{1} \\ &= 77.28.\end{aligned}$$

5. (23 points) Consider the following data representing the total number of votes for the office of President of the United States (in millions) for general elections from 1996 to 2012:

   96.3 105.5 122.3 131.3 129.0

(a) (5 points) Find the range.
Solution: Calculate

$$\text{range} = \max - \min = 131.3 - 96.3 = 35.0.$$

(b) (6 points) Find the mean.
Solution: Recall the sample mean of a sample of size n is given by

$$\overline{x} = \frac{\sum x}{n},$$

where x represents our data points. Calculate the sample mean

$$\begin{aligned}\overline{x} &= \frac{96.3 + 105.5 + 122.3 + 131.3 + 129.0}{5} \\ &= \frac{584.4}{5} \\ &= 116.88.\end{aligned}$$

(c) (6 points) Find the sample variance.
Solution: Recall that the sample variance of a sample of size n is

$$\text{SampleVariance} = s^2 = \frac{\sum(x - \overline{x})^2}{n-1},$$

where x represents our data points and $\overline{x}$ denotes the sample mean. So we compute

$$\begin{aligned}&\text{SampleVariance} \\ &= \frac{(96.3 - 116.88)^2 + (105.5 - 116.88)^2 + (122.3 - 116.88)^2 + (131.3 - 116.88)^2 + (129.0 - 116.88)^2}{5-1} \\ &= \frac{(-20.58)^2 + (-11.38)^2 + (5.42)^2 + (14.42)^2 + (12.12)^2}{4} \\ &= \frac{423.5364 + 129.5044 + 29.3764 + 207.9364 + 146.8944}{4} \\ &= \frac{937.248}{4} \\ &= 234.312.\end{aligned}$$

(d) (6 points) Find the sample standard deviation.
Solution: The sample standard deviation s is given by

$$s = \sqrt{\text{SampleVariance}}.$$

We use our value from part (c) and compute

$$\text{SampleStandardDeviation} = \sqrt{\text{SampleVariance}} = \sqrt{234.312} = 15.307.$$

6. (12 points) (a) (6 points) The mean monthly utility bill for a sample of households in a city is \$70, with a standard deviation of \$8. Between what two values does about 95% of the data lie? (Assume the data has a bell-shaped distribution)
Solution: By the "empirical rule", 95% of the data in a bell-shaped distribution lies within 2 standard deviations of the mean. That means the lower value is

$$\$70 - 2(\$8) = \$70 - \$16 = \$54,$$

and the upper value is

$$\$70 + 2(\$8) = \$70 + \$16 = \$86.$$

(b) (6 points) You are conducting a survey on the number of people per household in your region. From a sample with $n = 51$, the mean number of pets per household is 3 pets and the standard deviation is 1 pets. Using Chebychev's theorem, determine at least how many of the households have 1 to 5 pets.

Solution: The value 1 is two standard deivation from the mean (to the left) and the value 5 is two standard deviations from the mean (to the right). This means we should apply Chebychev's theorem with $k = 2$ to conclude that approximately

$$1 - \frac{1}{k^2} = 1 - \frac{1}{4} = \frac{3}{4} = 75\%$$

of the data lies between 1 and 5 pets. So the number of households with this many pets is 75% of 51, or

$$51(0.71) = 38.25,$$

so 38 or 39 of the houses have between 1 and 5 pets.

7. (21 points) Consider the data

51	54	55	56	57
58	59	59	60	60
61	62	63	63	70

(a) (6 points) Find the quartiles Q_1, Q_2, and Q_3.

Solution: The second quartile Q_2 is the median. Notice that the data is already ordered, and so the median will be the halfway point, which occurs at

$$Q_2 = 59.$$

Now Q_1 is the middle data point of the data to the left of Q_2 and has value

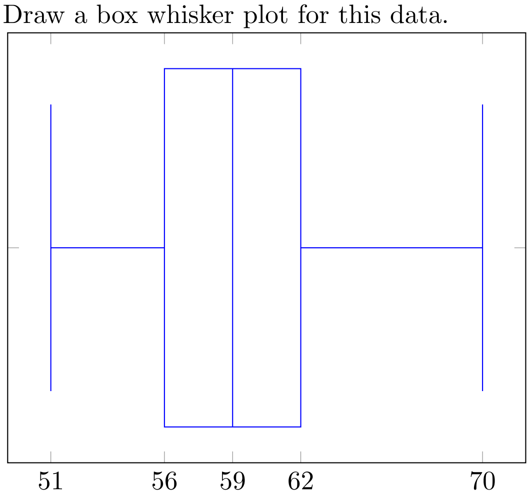
$$Q_1 = 56.$$

The third quartile Q_3 is the middle data point of the data to the right of Q_2 and has value

$$Q_3 = 62.$$

(b) (5 points) Draw a box whisker plot for this data.

Solution:



(c) (5 points) Find the IQR.
Solution: The IQR is given by

$$IQR = Q_3 - Q_1 = 62 - 56 = 6.$$

(d) (5 points) Identify outliers.
Solution: First compute

$$1.5IQR = 9$$

Outliers are data points that lie below

$$Q_1 - 1.5IQR = 56 - 9 = 47$$

or above

$$Q_3 + 1.5IQR = 62 + 9 = 71.$$

No such data points exist, so this data set has no outliers.

8. (5 points) The distribution of a set of data is approximately bell-shaped. The mean of the data is 18.7 with a standard deviation of 3.2. Find the z-score of the data point $x = 19.3$.
Solution: We are told the mean $\mu = 18.7$ and the standard deviation $\sigma = 3.2$. The z score is given by

$$\frac{x - \mu}{\sigma} = \frac{19.3 - 18.7}{3.2} = 0.1875.$$

9. (17 points) A probability experiment is carried out by flipping two coins and then rolling a 4-sided die.

(a) (5 points) How many outcomes are in the sample space?
Solution: There are 16:

First flip	Second flip	Die roll
H	H	1
H	H	2
H	H	3
H	H	4
H	T	1
H	T	2
H	T	3
H	T	4
T	H	1
T	H	2
T	H	3
T	H	4
T	T	1
T	T	2
T	T	3
T	T	4

(b) (6 points) How many outcomes are in the event of flipping two heads and then rolling a 3? What is the probability of this event?
Solution: There is exactly one outcome in this event. Its probability is $\frac{1}{16} = 0.0625$.

(c) (6 points) How many outcomes are in the event of the first coin flip being heads and rolling a number bigger than 2? What is the probability of this event?
Solution: There are four outcomes in this event, HH3, HT3, HH4, and HT4. The probability is $\frac{4}{16} = \frac{1}{4} = 0.25$.