MATH 2222HJ Spring 2016
Quiz 6 SOLUTION

Show all work clearly and in order, and circle your final answers.
Justify your answers algebraically whenever possible. Unjustified work may not receive full credit.

1. (*2 points*) Find $\int_C y\sin(z)\mathrm{d}s$ where C is the circular helix given by $\begin{cases} x = \cos(t) \\ y = \sin(t) \\ z = t \\ 0 \le t \le 2\pi \end{cases}$.

Solution: Our curve is parametrized as

$$\begin{cases} \vec{r}(t) = < \cos(t), \sin(t), t > \\ 0 \le t \le 2\pi. \end{cases}$$

Hence $\vec{r}'(t) = < -\sin(t), \cos(t), 1 >$ and

$$\|\vec{r}'(t)\| = \sqrt{\sin^2(t) + \cos^2(t) + 1} = \sqrt{2}.$$

Therefore compute

$$\begin{aligned} \int_C y\sin(z)\mathrm{d}s &= \sqrt{2}\int_0^{2\pi} \sin^2(t)\mathrm{d}t \\ &= \frac{\sqrt{2}}{2}\int_0^{2\pi} 1 - \cos(2t)\mathrm{d}t \\ &= \frac{1}{\sqrt{2}}\left[t - \frac{1}{2}\sin(t)\right]_0^{2\pi} \\ &= \sqrt{2}\pi. \end{aligned}$$

2. (*3 points*) Evaluate $\int_C \vec{F}(x,y,z)d\vec{r}$ where $\vec{F} = < y, z, x >$ and $C = C_1 \cup C_2$ is the union of two curves, where C_1 is the line segment from $(2,0,0)$ to $(3,4,5)$ and C_2 is the line segment from $(3,4,5)$ to $(3,4,0)$.
Solution: Parametrize C_1 as

$$\begin{cases} \vec{r}_1(t) = t < 3,4,5 > + (1-t) < 2,0,0 > = < t+2, 4t, 5t > \\ 0 \le t \le 1, \end{cases}$$

and parametrize C_2 as

$$\begin{cases} \vec{r}_2(t) = t < 3,4,0 > + (1-t) < 3,4,5 > = < 3, 4, 5-5t > \\ 0 \le t \le 1, \end{cases}$$

so we see

$$\vec{r}_1'(t) = < 1, 4, 5 >,$$

and

$$\vec{r}_2'(t) = < 0, 0, -5 > .$$

Hence compute

$$\begin{aligned}
\int_C < y,z,x > \mathrm{d}\vec{r} &= \int_{C_1} < y,z,x > \mathrm{d}\vec{r} + \int_{C_2} < y,z,x > \mathrm{d}\vec{r} \\
&= \int_0^1 < 4t, 5t, t+2 > \cdot < 1,4,5 > \mathrm{d}t + \int_0^1 < 4, 5-5t, 3 > \cdot < 0,0,-5 > \mathrm{d}t \\
&= \int_0^1 4t + 20t + 5t + 10\mathrm{d}t + \int_0^1 -15\mathrm{d}t \\
&= \int_0^1 29t + 10\mathrm{d}t - 15\int_0^1 1\mathrm{d}t \\
&= (\frac{29}{2} + 10) - 15 \\
&= \frac{19}{2}.
\end{aligned}$$