MATH 2222H&J Spring 2016
Quiz 1 SOLUTION

1. (*1 point*) What does the equation $x^2 + (y-1)^2 = 4$ describe in $\mathbb{R}^2$?
Solution: Recall that the equation $(x-a)^2 + (y-b)^2 = r^2$ describes the circle of radius r centered at (a, b). Therefore the equation in question describes the circle of radius 2 centered at $(0, 1)$.

2. (*2 points*) Let $\vec{a} =< 3, 1, 2 >$ and $\vec{b} =< -1, 4, 3 >$. Calculate $\|\vec{b}\|$, $\vec{a} - 7\vec{b}$, and $\|\vec{a} - 7\vec{b}\|$.
Solution: First compute

$$\|\vec{b}\| = \sqrt{(-1)^2 + 4^2 + 3^2} = \sqrt{1 + 16 + 9} = \sqrt{26}.$$

Now compute

$$\begin{aligned}\vec{a} - 7\vec{b} \quad &=< 3, 1, 2 > -7 < -1, 4, 3 > \\ &=< 3, 1, 2 > + < 7, -28, -21 > \\ &=< 3 + 7, 1 - 28, 2 - 21 > \\ &=< 10, -27, -19 > .\end{aligned}$$

Finally compute

$$\|\vec{a} - 7\vec{b}\| = \|< 10, -27, -19 >\| = \sqrt{10^2 + (-27)^2 + (-19)^2} = \sqrt{1190}.$$

3. (*2 points*) Let $P = (3, 4, -1)$ and $Q = (2, 2, 1)$ be points in $\mathbb{R}^3$. Find the vectors $\overrightarrow{PQ}$ (the vector from P to Q) and $\overrightarrow{QP}$ (the vector from Q to P). What relationship does $\overrightarrow{PQ}$ have to $\overrightarrow{QP}$?
Solution: First compute

$$\overrightarrow{PQ} = Q - P =< 2, 2, 1 > - < 3, 4, -1 >=< 2 - 3, 2 - 4, 1 - (-1) >=< -1, -2, 2 > .$$

Now compute

$$\overrightarrow{QP} = P - Q =< 3, 4, -1 > - < 2, 2, 1 >=< 3 - 2, 4 - 2, -1 - 1 >=< 1, 2, -2 > .$$

We see that $\overrightarrow{PQ} = -\overrightarrow{QP}$, i.e., it is the same vector with opposite direction.