MATH 2222HJ Spring 2016
Quiz 2 SOLUTION

Show all work clearly and in order, and circle your final answers.
Justify your answers algebraically whenever possible. Unjustified work may not receive full credit.

1. (*2 points*) Find a parametrization of the tangent line of the helix $\vec{H}(t) = \langle \cos(t), \sin(t), t \rangle$ at the point $\left(0, 1, \frac{\pi}{2}\right)$.

Solution: First note that the value of t_0 such that $\vec{H}(t_0) = \left\langle 0, 1, \frac{\pi}{2} \right\rangle$ is $t_0 = \frac{\pi}{2}$. Now find the tangent vector function

$$\vec{H}\,'(t) = \langle -\sin(t), \cos(t), 1 \rangle .$$

Thus the tangent vector at the point in question is $\vec{H}\,'\left(\frac{\pi}{2}\right) =< -1, 0, 1 >$. The tangent line we are seeking is the line that goes through the point $\left(0, 1, \frac{\pi}{2}\right)$ parallel to the tangent vector $< -1, 0, 1 >$, hence the tangent line is given by

$$\vec{r}(t) = \left\langle 0, 1, \frac{\pi}{2} \right\rangle + t < -1, 0, 1 > .$$

2. (*3 points*) Find the arc length of the curve $\begin{cases} \vec{r}(t) = \langle t, -\log(\cos(t)) \rangle \\ -\frac{\pi}{4} \leq t \leq \frac{\pi}{4}. \end{cases}$

Note: you may find the trigonometric identity $\tan^2(t) + 1 = \sec^2(t)$ and the following integral formula useful:

$$\int \sec(t)\mathrm{d}t = \log(\tan(t) + \sec(t)) + C$$

Solution: Compute the tangent vector function $\vec{r}'(t) =< 1, \tan(t) >$. Now compute the arc length as follows:

$$\begin{aligned}
L &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \| < 1, \tan(t) > \| dt \\
&= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{1 + \tan^2(t)}dt \\
&= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sqrt{\sec^2(t)}dt \\
&= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec(t)dt \\
&= \log(\tan(t) + \sec(t))\Big|_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \\
&= \log\left(\tan\left(\frac{\pi}{4}\right) + \sec\left(\frac{\pi}{4}\right)\right) - \log\left(\tan\left(-\frac{\pi}{4}\right) + \sec\left(-\frac{\pi}{4}\right)\right) \\
&= \log(1 + \sqrt{2}) - \log(-1 + \sqrt{2}) \\
&= \log\left(\frac{\sqrt{2} + 1}{\sqrt{2} - 1}\right) \\
&= \log(3 + 2\sqrt{2}),
\end{aligned}$$

where on the last line we rationalized the denominator.