Math 2222 Test #4 Your Name:_____

Dec. 7, 2015 Instructor's Name and Class Time:_____

Directions:

(1) All cell phones and other electronic noisemaking devices must be turned completely off and put away for the duration of the exam.
(2) No calculators, books, or other materials are permitted.
(3) Show **ALL** your work! Correct answer which are not properly justified will not receive full credit.
(4) Failure to follow directions specific to a problem will result in the loss of points.
(5) Write your answer in the space provided, if any. Otherwise, circle or box your answer.
(6) If you work a problem two different ways, clearly indicate which one you want us to grade, preferably by crossing out the one you do not want us to look at. If you use two methods, one of which is wrong, and neither method is crossed out, you will not receive full credit.
(7) Answers must be exact (like $\sqrt{2}$) not approximate (like 1.414), unless a problem specifically indicates otherwise.
(8) Simplify where appropriate. Quantities such as $\sqrt{9}$ and $\cos \pi$ should be calculated.
(9) If you need extra room, you may used the back of the **previous** page. However, you must indicate you are doing so by clearly writing "BPP" on the relevant problem.
(10) This packet has seven sheets of paper, including this cover page and the formula sheet at the end of the exam. You may remove the formula sheet from the packet, but do NOT remove the staple or remove any other sheet from this packet.
(11) Once this exam begins, you will have 50 minutes to complete your solutions.

DO NOT OPEN THIS EXAM UNTIL TOLD TO DO SO

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1. (14 pts.) Let C be the part of the curve $y = \frac{1}{3}x^3$ from $(1, 1)$ to $(2, \frac{8}{3})$. Evaluate the line integral $\displaystyle\int_C y\, ds$.

2. (15 pts.) Find the work done by the force field $\mathbf{F}(x, y, z) = \langle \dfrac{1}{1+z^2}, e^{xz+y}, yz \rangle$ in moving a particle along the straight line segment from $(2, 6, 0)$ to $(1, 6, 1)$.

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3. (14 pts.) Evaluate the line integral $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where C is the curve that starts from the origin, along the y-axis to the point $(0, 2)$, counterclockwise along the circle $x^2 + y^2 = 4$ to the point $(-2, 0)$, then back to the origin along the x-axis and $\mathbf{F}(x, y) = \langle x^{2015}e^x - x^2y, xy^2 - \sin(\tan(\pi y^{10}))\rangle$.

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4. (a) (16 pts.) Evaluate $\iint_S x\,dS$, where the surface S is given by $\mathbf{r}(u,v) = \langle u, u\cos v, u\sin v\rangle$ for $0 \le u \le 3$ and $0 \le v \le 2\pi$.

(b) (4 pts.) Identify the type of surface of which the given surface is a part (figuring out the relationship among x, y, and z will give partial credit).

Answer:_____

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5. (12 pts.) Set up, but **DO NOT EVALUATE**, a double (iterated) integral to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = \langle xz, -z, xy \rangle$ and S is the part of the surface $z = xe^y$ (with upward orientation) lying above the triangle in the xy-plane with vertices $(0, 0)$, $(2, 2)$, and $(2, -2)$. Make sure to do all vector operations and substitute as appropriate (but you do not need to do simplifying algebra).

6. Let $\mathbf{F}(x, y, z) = \langle e^{xy}, yz, \frac{x}{z} \rangle$.

(a) (6 pts.) Find the divergence of $\mathbf{F}$.

(b) (6 pts.) Find the curl of $\mathbf{F}$.

(c) (3 pts.) Is $\mathbf{F}$ conservative? Explain briefly.

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7. (10 pts.) Let $\phi(x, y, z) = xy + x\sin(\pi z) + ze^y$ and let $\mathbf{F}(x, y, z) = \nabla\phi$. Evaluate the line integral $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the curve $\mathbf{r}(t) = \langle 3 - 2t, 5t^{2015}, (1 - t)/\sqrt{4 + t}\rangle$, $0 \le t \le 1$.

<u>Formula Sheet</u>

$\sin 2\theta = 2\sin\theta\cos\theta$; $\cos 2\theta = \cos^2\theta - \sin^2\theta$; $\sin^2\theta = \frac{1-\cos 2\theta}{2}$; $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$\int \frac{dx}{1+x^2} = \tan^{-1} x + C; \qquad \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$$

$x = r\cos\theta$; $y = r\sin\theta$; $z = z$; $x^2 + y^2 = r^2$

$x = \rho\sin\phi\cos\theta$; $y = \rho\sin\phi\sin\theta$; $z = \rho\cos\phi$; $x^2 + y^2 + z^2 = \rho^2$

$dx\,dy\,dz = r\,dz\,dr\,d\theta = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

$\int_C \mathbf{F}\cdot\,d\mathbf{r} = \int_a^b \mathbf{F}\cdot\mathbf{r}'(t)\,dt$; $\iint\limits_S f\,dS = \iint\limits_R f|\mathbf{t}_u \times \mathbf{t}_v|\,dA$; $\iint\limits_S \mathbf{F}\cdot d\mathbf{S} = \iint\limits_R \mathbf{F}\cdot(\mathbf{t}_u \times \mathbf{t}_v)\,dA$

$$\oint_C f\,dx + g\,dy = \iint\limits_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right) dA \text{ (circulation form)}$$

$$\oint_C f\,dy - g\,dx = \iint\limits_R \left(\frac{\partial f}{\partial x} + \frac{\partial g}{\partial y}\right) dA \text{ (flux form)}$$

$$\oint_C x\,dy = -\oint_C y\,dx = \frac{1}{2}\oint_C (x\,dy - y\,dx)$$

$\operatorname{curl}\mathbf{F} = \nabla\times\mathbf{F}$; $\operatorname{div}\mathbf{F} = \nabla\cdot\mathbf{F}$; $\mathbf{t}_x \times \mathbf{t}_y = \langle -z_x, -z_y, 1\rangle$