Math 2222 Test #2 Your Name:____________________________

Mar. 16, 2015 Instructor's Name and Class Time:____________________

Directions:

(1) All cell phones and other electronic noisemaking devices must be turned completely off and put away for the duration of the exam.
(2) No calculators, books, or other materials are permitted.
(3) Show **ALL** your work! Correct answer which are not properly justified will not receive full credit.
(4) Failure to follow directions specific to a problem will result in the loss of points.
(5) Write your answer in the space provided, if any. Otherwise, circle or box your answer.
(6) If you work a problem two different ways, clearly indicate which one you want us to grade, preferably by crossing out the one you do not want us to look at. If you use two methods, one of which is wrong, and neither method is crossed out, you will not receive full credit.
(7) Answers must be exact (like $\sqrt{2}$) not approximate (like 1.414), unless a problem specifically indicates otherwise.
(8) Simplify where appropriate. Quantities such as $\sqrt{9}$ and $\cos\pi$ should be calculated.
(9) If you need extra room, you may used the back of the previous page. However, you must indicate you are doing so by clearly writing "BPP" on the relevant problem.
(10) This packet has six sheets of paper, including this cover page. Do NOT remove the staple or remove any sheet from this packet.
(11) Once this exam begins, you will have 50 minutes to complete your solutions.

$$D(x,y) = f_{xx}f_{yy} - (f_{xy})^2$$

If $D(a,b) > 0$ and $f_{xx}(a,b) > 0$, then f has a local minimum at (a,b).
If $D(a,b) > 0$ and $f_{xx}(a,b) < 0$, then f has a local maximum at (a,b).

$$\nabla f = \lambda \nabla g$$

DO NOT OPEN THIS EXAM
UNTIL TOLD TO DO SO

1. (10 pts.) Let $f(x,y) = \begin{cases} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}, & (x,y) \neq (0,0) \\ b, & (x,y) = (0,0). \end{cases}$

Find the value of b, if any, that makes $f(x,y)$ continuous at $(0,0)$, or explain why no such b exists.

2. (10 pts.) A quadric surface has cross-sections that are parabolas for constant x and for constant z, ellipses for constant $y > 0$, a single point for $y = 0$, and the empty set for $y < 0$. Circle which of the following could be an equation for the surface, and also identify the surface.

a. $\dfrac{x^2}{4} + \dfrac{z^2}{9} = y^2$

b. $\dfrac{x^2}{9} + y - \dfrac{z^2}{4} = 0$

c. $\dfrac{x^2}{4} + y^2 + \dfrac{z^2}{9} = 1$

d. $\dfrac{x^2}{4} + \dfrac{z^2}{9} = y$

Name of surface________________________________

3. (12 pts.) Find an equation of the tangent plane to the surface $z = \cos(2x) + 3xy$ at the point $(\pi, \frac{1}{\pi}, 4)$.

4. (10 pts.) **Use the chain rule** to find the derivative $\dfrac{dz}{dt}$, given that $z = xe^{xy}$, $x = \sin t$, and $y = t^2 + 1$. Write your answer in terms of t only. It is not necessary to do simplifying algebra.

5. For all parts of this problem, let $f(x, y) = 6\sqrt{x + 2y^2}$. (Part (d) is on the next page.)

(a) (6 pts.) Find the gradient $\nabla f(x, y)$.

(b) (10 pts.) Find the directional derivative of f in the direction of the vector $\mathbf{v} = \langle -1, 1 \rangle$ at the point $P(1, 2)$.

(c) (6 pts.) Find the maximum rate of change of f at $P(1, 2)$ and the direction in which it occurs.

Max rate of change: ____________________

Direction of max rate of change: ____________________

5(d) (continued from previous page) (4 pts.) Find a direction (you don't have to write it as a unit vector) in which the directional derivative of f is 0 at the point $(1, 2)$. What relationship does this direction have to the gradient at that point?

Direction vector:____________________________

Relationship to gradient:____________________________

6. (12 pts.) The function $f(x, y) = 3x^2 + 6xy + 2y^3 + 12x - 24y$ has critical points $(0, -2)$ and $(-5, 3)$ (you do not need to verify this). Classify each of these critical points as the location of a local maximum, local minimum, saddle point, or none of the above, or state that there is not enough information to classify the critical point.

7. (9 pts.) Find all critical points of the function $f(x, y) = x^3 + 6xy + 3y^2 - 9x$. **You do not need to classify these critical points.**

8. (11 pts.) Use the method of Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = x - 3y - 1$ subject to the constraint $x^2 + 3y^2 = 16$.