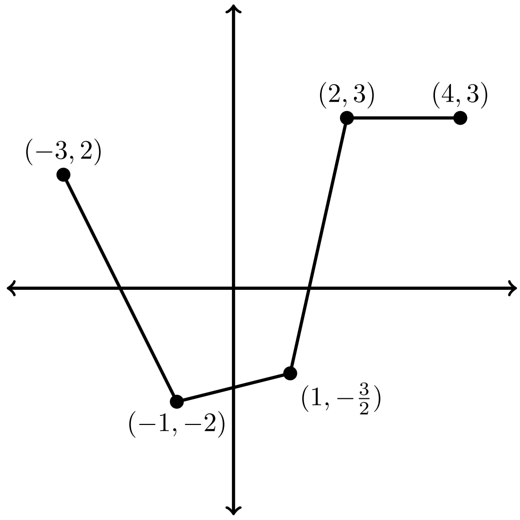
MATH 1112 - EXAM 2 - FALL 2016
SOLUTION

Friday 14 October 2016
Instructor: Tom Cuchta

Instructions:

- Show all work, clearly and in order, if you want to get full credit. If you claim something is true **you must show work backing up your claim**. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Justify your answers algebraically whenever possible to ensure full credit.
- Circle or otherwise indicate your final answers.
- Please keep your written answers brief; be clear and to the point.
- Good luck!

1. (12 points) For the following function, state where it is increasing, decreasing, and constant.



Increasing: $\underline{(-1, 2)}$ Decreasing: $\underline{(-3, -1)}$ Constant: $\underline{(2, 4)}$

2. (8 points) Let

$$f(x) = \begin{cases} x + 7, & -\infty < x < 5 \\ -2x + 1, & 5 \leq x < \infty \end{cases}$$

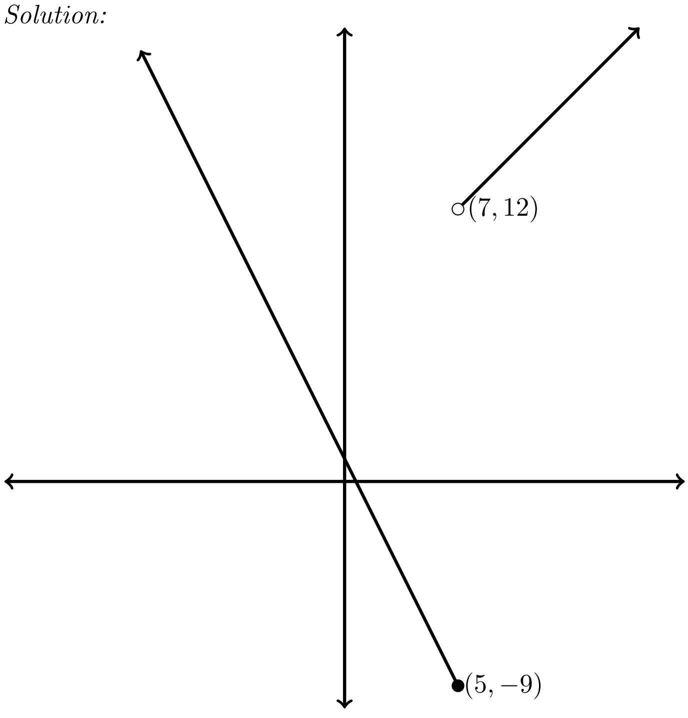
(a) (2 points) What is the value of $f(-10)$?
Solution: Since $-\infty < -10 < 5$,

$$f(-1) = -10 + 7 = -3.$$

(b) (2 points) What is the value of $f(2)$?
Solution: Since $-\infty < 2 < 5$,

$$f(2) = 2 + 7 = 9.$$

(c) (4 points) Graph $y = f(x)$.
Solution:



3. (10 points) Let $f(x) = \sqrt{x}$ and $g(x) = 3x - 2$.

(a) (3 points) Compute $(f + g)(2)$
Solution: Compute

$$(f+g)(2) = f(2) + g(2) = \sqrt{2} + (3 \cdot 2 - 2) = \sqrt{2} + 4.$$

(b) (3 points) Compute $(f - g)(x)$
Solution: Compute

$$(f-g)(x) = f(x) - g(x) = \sqrt{x} - (3x - 2) = \sqrt{x} - 3x + 2.$$

(c) (4 points) What is the domain of $\left(\frac{f}{g}\right)$?
Solution: Notice that

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{x}}{3x-2}.$$

The top forces $x \geq 0$ (which could be written as the interval $[0, \infty)$ and we must also remove any values of x that makes $g(x) = 0$, i.e. we must remove x such that $3x - 2 = 0$, i.e. we must remove x such that $x = \frac{2}{3}$. Therefore the domain is

$$\left[0, \frac{2}{3}\right) \bigcup \left(\frac{2}{3}, \infty\right).$$

4. (13 points) Suppose $f(x) = x^2 + 2$ and $g(x) = \sqrt{x-3}$.

(a) (3 points) Compute $(f \circ g)(5)$
Solution: First note that

$$g(5) = \sqrt{5-3} = \sqrt{2}.$$

Now compute

$$(f \circ g)(5) = f(g(5)) = f(\sqrt{2}) = (\sqrt{2})^2 + 2 = 2 + 2 = 4.$$

(b) (3 points) Compute $(f \circ g)(x)$
Solution: Compute

$$f(g(x)) = f(\sqrt{x-3}) = \left(\sqrt{x-3}\right)^2 + 2 = x - 3 + 2 = x - 1.$$

(c) (3 points) Compute $(g \circ f)(1)$
Solution: First note that

$$f(1) = 1^2 + 2 = 1 + 2 = 3.$$

Now compute

$$(g \circ f)(1) = g(f(1)) = g(3) = \sqrt{3-3} = \sqrt{0} = 0.$$

(d) (4 points) What is the domain of $(f \circ g)(x)$?
Solution: Note that

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x-3}).$$

Since f is a polynomial, it will not restrict the domain. Since $g(x) = \sqrt{x-3}$ we must have $x - 3 \geq 0$, i.e. $x \geq 3$.
note: If the computation is continued, we see that $f(g(x)) = x - 1$, which is a polynomial. This does not mean the domain of $f \circ g$ is $\mathbb{R}$ – we have to use the restriction given by g since g is part of the composition.

5. (9 points) Test the following for any symmetry. Circle yes or no for each option. *Failure to show the work behind your decision will cause you to lose points.*

$$3x^2 - x + y^2 = 0$$

Solution: To test for x-axis symmetry, replace y with $-y$ and compare to the original:

$$3x^2 - x + (-y)^2 = 0.$$

But $(-y)^2 = (-y)(-y) = y^2$, so this is equivalent to

$$3x^2 - x + y^2 = 0,$$

which matches the original. Hence we have x-axis symmetry.
To test for y-axis symmetry, replace x with $-x$ and compare to the original:

$$3(-x)^2 - (-x) + y^2 = 0.$$

But $(-x)^2 = x^2$ so we have

$$3x^2 + x + y^2 = 0,$$

which does not match the original, so we do not have y-axis symmetry.
To test for origin symmetry, replace x with $(-x)$ and replace y with $(-y)$ to get

$$3(-x)^2 - (-x) + (-y)^2 = 0,$$

which is equivalent to

$$3x^2 + x + y^2 = 0,$$

which does not match the original. Therefore it does not have origin symmetry.

6. (6 points) Determine if the following function is even, odd, or neither:

$$f(x) = 3x^2 - 3$$

Failure to show the reasoning behind your decision will cause you to lose points.
Solution: Compute

$$f(-x) = 3(-x)^2 - 3 = x^2 - 3 = f(x),$$

therefore f is even.

7. (16 points) Consider $y = -(x-2)^2$. Fill in the following information about transformations on the function. If a specific transformation is not used, state so.

(a) (2 points) Base Function: x^2

(b) (2 points) Reflections: vertical reflection

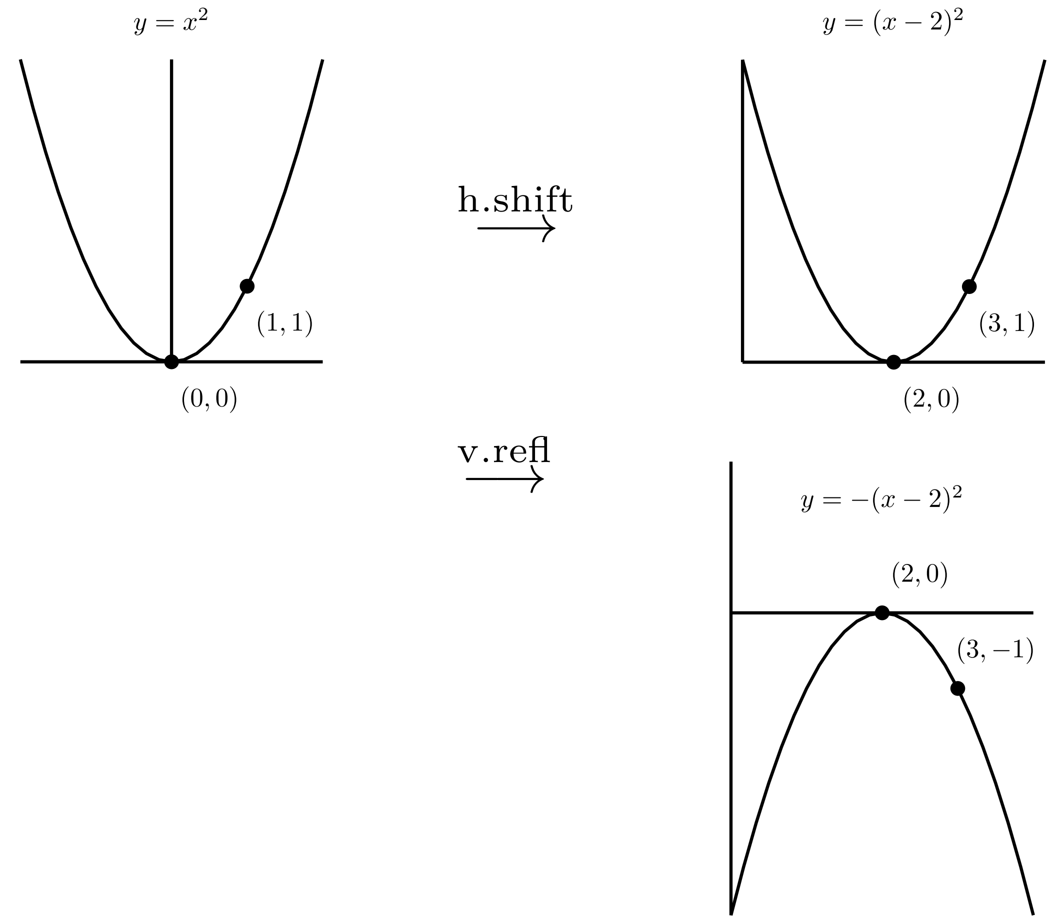
(c) (2 points) Horizontal Stretch: none

(d) (2 points) Vertical Stretch: none

(e) (2 points) Horizontal Shift: 2 to the right

(f) (2 points) Vertical Shift: none

(g) (4 points) Graph $y = -(x-2)^2$ with appropriate labeling.
Solution:



8. (16 points) Let $g(x) = 2\sqrt{x+3} - 2$. Fill in the following information about transformations on the function. If a specific transformation is not used, state so.

   (a) (2 points) Base Function: $\sqrt{x}$

   (b) (2 points) Reflections: none

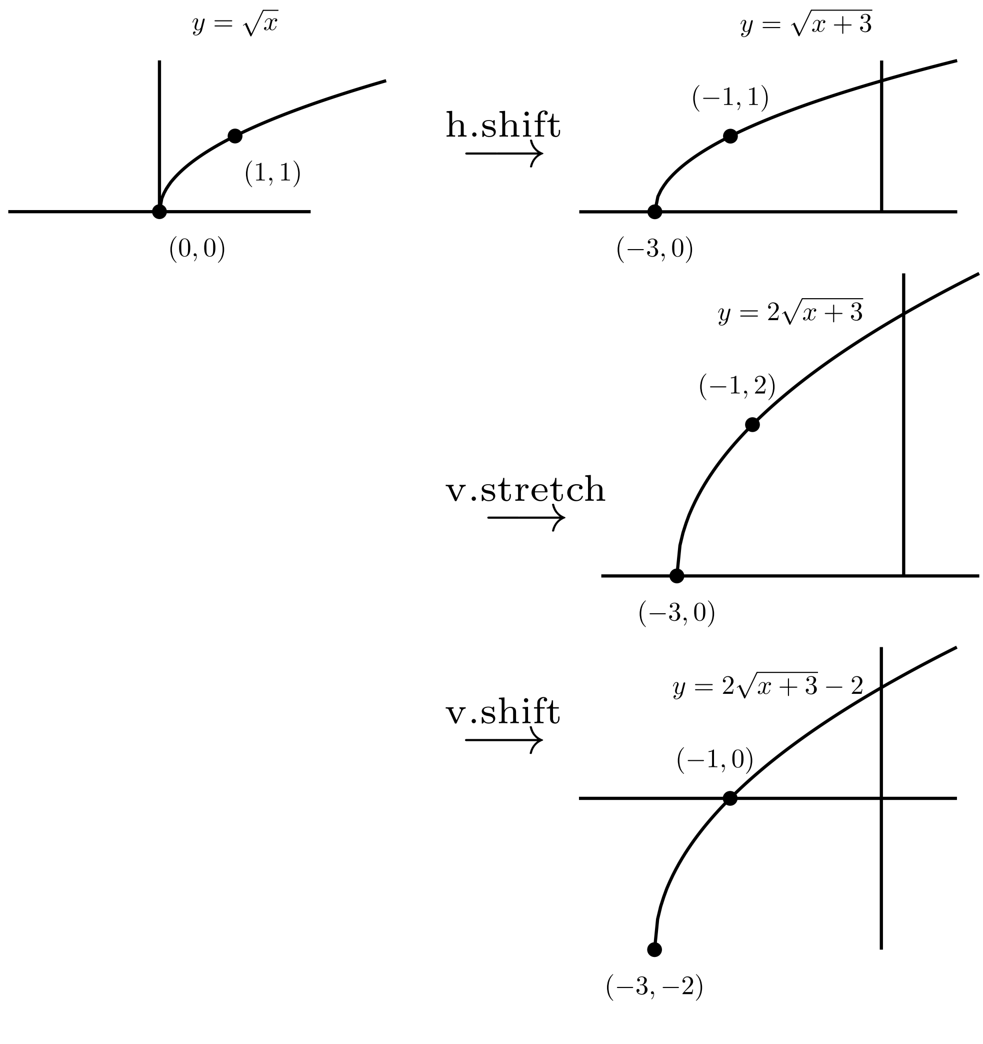
   (c) (2 points) Horizontal Stretch: none

   (d) (2 points) Vertical Stretch: none

   (e) (2 points) Horizontal Shift: left 3

   (f) (2 points) Vertical Shift: down 2

   (g) (4 points) Graph $y = 2\sqrt{x+3} - 2$ with appropriate labeling.



9. (10 points) Do the follow two problems.

(a) (5 points) Hooke's law states that the distance d that a spring will stretch is directly proportional to the mass m of an object hanging from the spring. A 2 kg mass stretches the spring 40 cm. How far will a 5 kg mass stretch the spring?

Solution: The first sentence gives us the equation

$$d = km.$$

To find k we plug in $d = 40$ and $m = 2$ to get

$$40 = 2k,$$

or

$$20 = k.$$

This means the equation is actually

$$d = 20m.$$

Now plug in $m = 5$ to find the requested value for d:

$$d = 20(5) = 100.$$

(b) (5 points) The current I (in amperes) in a circuit is inversely proportional to its resistance R (in ohms). When the current in a given circuit is 30 amperes, the resistance is 8 ohms. Find the resistance in the circuit when the current is 2 amperes.

Solution: The first sentence gives us the equation

$$I = \frac{k}{R}.$$

To find k, we plug in $I = 30$ and $R = 8$ to get

$$30 = \frac{k}{8},$$

or

$$k = 240.$$

Therefore the equation is

$$I = \frac{240}{R}.$$

Now plug in $I = 2$ to find the requested value for R:

$$2 = \frac{240}{R},$$

so

$$R = \frac{240}{2} = 120.$$