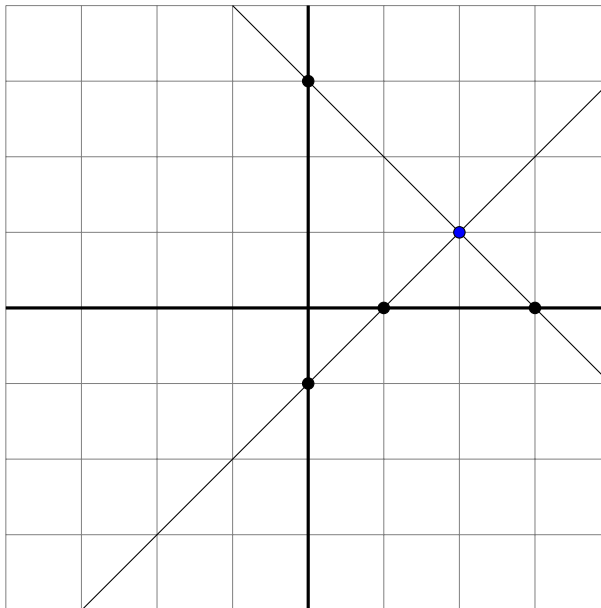


Quiz 21

1. Solve the following system of equations by graphing each line (by finding two solutions to each and connecting the dots) and finding the intersection point in the given graph: $\begin{cases} x + y = 3 & (i) \\ x - y = 1 & (ii) \end{cases}$

Solution: First we will plot the lines by finding two solutions and connecting the dots (so our lines are entirely accurate). To draw (i), first plug in $x = 0$ into (i) to get $y = 3$ — hence $(0, 3)$ is a solution of (i). Also plug in $y = 0$ to get $x = 3$ giving $(3, 0)$ a solution of (i). Plot these points and connect the dots to draw (i).

To draw (ii), first plug $x = 0$ into (ii) to get $-y = 1$ or $y = -1$ — this means $(0, -1)$ is a solution of (ii). Plug $y = 0$ into (ii) to get $x = 1$ — this means $(1, 0)$ is a solution of (ii). Plot these points and connect the dots to draw (ii).



From the intersection of the lines, we see the solution of the system is the point $(2, 1)$.

2. Solve the following system of equation in any way you choose:

$$\begin{cases} x - y = 2 & (i) \\ x - y = 5 & (ii). \end{cases}$$

Solution: Solve (i) for x to get

$$x = 2 + y.$$

Plug this into (ii) to get

$$(2 + y) - y = 5,$$

or

$$2 = 5,$$

which is **always false**. Therefore there is **no solution** to this system.

3. Solve the following system of equations in any way you choose:

$$\begin{cases} 2x + 3y = 4 & (i) \\ x - 5y = 7 & (ii). \end{cases}$$

Solution: Solve (ii) for x :

$$x = 7 + 5y.$$

Plug this into (i) to get

$$2(7 + 5y) + 3y = 4.$$

Distribute the 2 to get

$$14 + 10y + 3y = 4.$$

Simplify by combining like terms to get

$$14 + 13y = 4.$$

Subtract 14 to get

$$13y = -10.$$

Finally divide by 13 to see $y = -\frac{10}{13}$. To get x plug this value into either (i) or (ii) – we will plug into (ii) – to get

$$x - 5 \left(-\frac{10}{13} \right) = 7,$$

or equivalently

$$x + \frac{50}{13} = 7.$$

This yields

$$x = 7 - \frac{50}{13} = \frac{91}{13} - \frac{50}{13} = \frac{41}{13}.$$

Therefore the solution of the system is $\left(\frac{41}{13}, -\frac{10}{13} \right)$.

4. Solve the following system of equations in any way you choose:

$$\begin{cases} 3x - 2y = 1 & (i) \\ -6x + 4y = -2 & (ii). \end{cases}$$

Solution: First we will solve (i) for x . Add $2y$ to get

$$3x = 1 + 2y$$

and divide by 3 to get

$$x = \frac{1}{3} + \frac{2}{3}y.$$

Plug this into (ii) to get

$$-6 \left(\frac{1}{3} + \frac{2}{3}y \right) + 4y = -2.$$

Distribute the -6 into the sum to get

$$-\frac{6}{3} - \frac{12}{3}y + 4y = -2,$$

or equivalently

$$-2 - 4y + 4y = -2,$$

or equivalently

$$-2 = -2.$$

This equation is **always true** and so this system has infinitely many solutions (so to write them down would require us to basically write (i) or (ii) again).