Problem: A Colombian coffee is worth $\frac{\$11}{\text{pound}}$ and a Turkish coffee is worth $\frac{\$14}{\text{pound}}$. You want to make 360 pounds of a mixture that is worth $\frac{\$12}{\text{pound}}$. How many pounds of each type should you use?

Solution: Let x represent the number of pounds of Colombian coffee and let y represent the number of pounds of Turkish coffee. This means that

$$\left(\frac{\$11}{\cancel{\text{pound}}}\right)(x\ \cancel{\text{pounds}}) = \$11x$$

and

$$\left(\frac{\$12}{\cancel{\text{pound}}}\right)(y\ \cancel{\text{pounds}}) = \$12y.$$

Our goal, when taking the total of the Colombian and Turkish coffee in the mix, is to have $\frac{\$12}{\text{pound}}$, we must have

$$\left(\frac{\$12}{\cancel{\text{pound}}}\right)((x+y)\ \cancel{\text{pounds}}) = \$12(x+y).$$

Therefore we are led to the following system of equations:

$$\begin{cases} x & +y & = 360 & (i) \\ 11x & +14y & = 12(x+y) & (ii) \end{cases}$$

Let us start with (ii). First we distribute the 12 into the sum to get

$$11x + 14y = 12x + 12y.$$

Subtract $12x$ and $12y$ from both sides to get

$$-x + 2y = 0.$$

Therefore add x and divide by 2 to get

$$y = \frac{x}{2}.$$

Plug this into equation (i) to get

$$x + \frac{x}{2} = 360.$$

Simplify the left-hand side to get

$$\frac{3x}{2} = 360.$$

Divide by 3 to get

$$\frac{x}{2} = 120.$$

Multiply by 2 to get

$$x = 120 \text{ pounds}.$$

Plugging this into (i) to get

$$120 + y = 360.$$

Subtract by 120 to get

$$y = 360 - 120 = 240 \text{ pounds}.$$

Therefore we should mix 120 pounds of Colombian coffee with 240 pounds of Turkish coffee.

Problem: Juice A is 15% orange juice and Juice B is 10% orange juice. How many liters of each type of juice be mixed to get 5 liters of 12% orange juice?
Solution: Let x represent the number of liters of Juice A and let y represent the number of liters of Juice B. To say that Juice A is 15% orange juice means that

$$\frac{15}{100}\frac{\text{liters of orange juice}}{\text{liters of Juice A}}.$$

So this means that we will consider

$$\left(\frac{15}{100}\frac{\text{liters of orange juice}}{\cancel{\text{liters of Juice A}}}\right)\left(x\,\cancel{\text{liters of Juice A}}\right) = \frac{15}{100}x \text{ liters of orange juice.}$$

To say that Juice B is 10% orange juice means that

$$\frac{10}{100}\frac{\text{liters of orange juice}}{\text{liters of Juice B}},$$

and so we would say that

$$\left(\frac{10}{100}\frac{\text{liters of orange juice}}{\cancel{\text{liters of Juice B}}}\right)\left(y\,\cancel{\text{liters of orange juice}}\right) = \frac{10}{100}y \text{ liters of orange juice.}$$

To seek a mixture of Juice A and Juice B which is 12% orange juice means we want the total amount of our mixture, $(x+y)$ liters (of the mixture), to be 12% orange juice, in other words,

$$\left(\frac{12}{100}\frac{\text{liters of orange juice}}{\cancel{\text{liters of the mixture}}}\right)\left((x+y)\,\cancel{\text{liters of orange juice}}\right) = \frac{12}{100}(x+y) \text{ liters of orange juice.}$$

Now the requirements of our problem that we have 5 total liters and the mixture must be 12% orange juice yields the following system of equations:

$$\begin{cases} x & +y & = 5 & (i) \\ \frac{15}{100}x & +\frac{10}{100}y & = \frac{12}{100}(x+y) & (ii). \end{cases}$$

We will start with equation (ii). First distribute $\frac{12}{100}$ into the sum to get

$$\frac{15}{100}x + \frac{10}{100}y = \frac{12}{100}x + \frac{12}{100}y.$$

Now subtract $\frac{12}{100}x$ and $\frac{12}{100}y$ to get

$$\frac{3}{100}x - \frac{2}{100}y = 0.$$

Now add $\frac{2}{100}y$ to both sides to get

$$\frac{3}{100}x = \frac{2}{100}y.$$

Multiply both sides by 100 to get

$$3x = 2y.$$

Therefore we see that $x = \frac{2}{3}y$. Now plug this into equation (i) to get

$$\frac{2}{3}y + y = 5.$$

Combine like terms on the left-hand side to get

$$\frac{5}{3}y = 5.$$

Therefore

$$y = \frac{15}{5} = 3.$$

We may plug this value of y into the equation $x = \frac{2}{3}y$ to get

$$x = \frac{2}{3}(3) = 2.$$

Therefore we should combine 2 liters of Juice A with 3 liters of Juice B to get a mixture containing 12% orange juice.