MATH 2222E Fall 2015
Quiz 1

SOLUTION

Show all work clearly and in order, and circle your final answers.
Justify your answers algebraically whenever possible. Unjustified work may not receive full credit.
You have 15 minutes to complete this 5 point quiz.

1. (*2 points*) Find an equation for the tangent line of the curve $\vec{r}(t) = \langle \cos(t), \sin(t), t \rangle$ when $t = \frac{\pi}{4}$.
Solution: Recall that the equation of a line is of the form

$$\vec{L}(t) = \vec{r}_0 + t\vec{v},$$

where $\vec{r}_0$ denotes the position vector of a point on the line and $\vec{v}$ is a vector parallel to the line. At $t = \frac{\pi}{4}$ we see that

$$\vec{r}\left(\frac{\pi}{4}\right) = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\pi}{4} \right\rangle,$$

which gives us the point $\vec{r}_0 = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\pi}{4} \right\rangle$. Now compute the derivative:

$$\frac{d\vec{r}}{dt}(t) = \langle -\sin(t), \cos(t), 1 \rangle .$$

Thus tangent vector at $t = \frac{\pi}{4}$ is given by

$$\frac{d\vec{r}}{dt}\left(\frac{\pi}{4}\right) = \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1 \right\rangle .$$

We take our parallel vector $\vec{v}$ to be $\vec{v} = \left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1 \right\rangle$. Therefore we have derived the equation for the tangent line:

$$\vec{L}(t) = \left\langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, \frac{\pi}{4} \right\rangle + t\left\langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 1 \right\rangle = \left\langle \frac{\sqrt{2}}{2} - t\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} + t\frac{\sqrt{2}}{2}, \frac{\pi}{4} + t \right\rangle .$$

2. (*3 points*) Suppose that $\vec{a}(t) =< 0, 0, -32 >$. If a projectile is launched from initial position $\vec{r}(0) =< 0, 0, 0 >$ and initial velocity $\vec{v}(0) =< 1, 2, 3 >$, find its position function $\vec{r}(t)$ (assume there is no air resistance on the projectile).
Solution: We compute the velocity $\vec{v}(t)$ by integrating the acceleration $\vec{a}(t)$:

$$\vec{v}(t) = \int \vec{a}(t)dt =< 0, 0, -32t > +\vec{c}.$$

To determine $\vec{c}$ we apply the initial condition $\vec{v}(0) =< 1, 2, 3 >$ and see

$$< 1, 2, 3 >= \vec{v}(0) =< 0, 0, -32(0) > + < c_1, c_2, c_3 >,$$

and hence $\vec{c} =< 1, 2, 3 >$ and our velocity is

$$\vec{v}(t) =< 0, 0, -32t > + < 1, 2, 3 >=< 1, 2, -32t + 3 > .$$

To find the position function $\vec{r}(t)$ we integrate the velocity:

$$\vec{r}(t) = \int \vec{v}(t)dt =< t, 2t, -16t^2 + 3t > +\vec{d}.$$

To determine $\vec{d}$ we apply the initial condition $\vec{r}(0) =< 0, 0, 0 >$ and see

$$< 0, 0, 0 >= \vec{r}(0) =< 0, 2(0), -16(0^2) + 3(0) > + < d_1, d_2, d_3 >,$$

and so $\vec{d} =< 0, 0, 0 >$. Therefore the position function of the projectile is

$$\vec{r}(t) =< t, 2t, -16t^2 + 3t > .$$