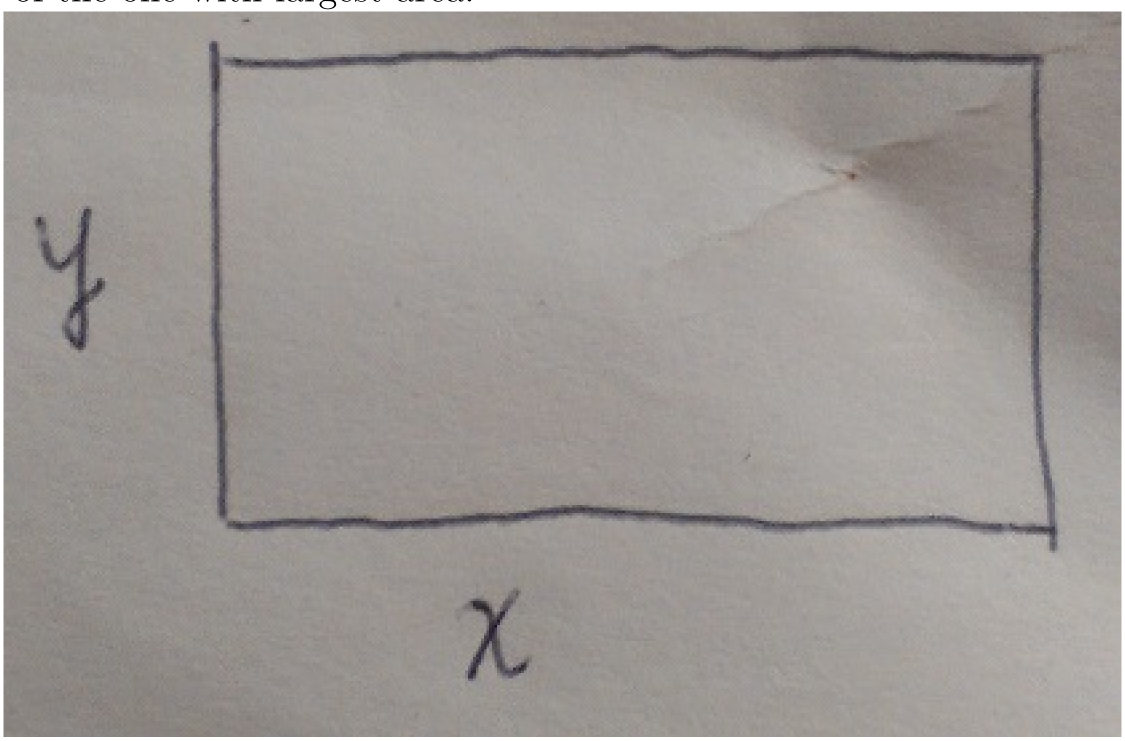
MATH 14-3D2 Spring 2014
Quiz 7 Name: SOLUTIONS

Show all work clearly and in order (on this sheet or an attached sheet) and circle your final answers.
Justify your answers algebraically whenever possible. Work without justification may not receive credit.
You have 25 minutes to take this 10 point quiz.

1. (*5 points*) Of all rectangles with a fixed perimeter P_0, give the dimensions of the one with largest area.



Solution:
Let x be a variable describing the width of such a rectangle and let y describe the height. We can express the P_0 in terms of x and y by

$$P_0 = 2x + 2y.$$

Solve this equation for y: $y = \frac{P_0}{2} - x$. The area of a rectangle with these dimensions is

$$\text{Area} = xy = x\left(\frac{P_0}{2} - x\right) = \frac{P_0}{2}x - x^2.$$

So we must optimize the function $A(x) = \frac{P_0}{2}x - x^2$. So $A'(x) = \frac{P_0}{2} - 2x$. Compute its critical point by solving $A'(x) = 0$, or equivalently

$$\frac{P_0}{2} - 2x = 0.$$

Its solution is $x = \frac{P_0}{4}$. Now we must prove that this x-value is a maximum – we will do so with the second derivative test. So compute $A''(x) = -2$. Thus $A''\left(\frac{P_0}{4}\right) = -2 < 0$ and by the second derivative test we conclude that $x = \frac{P_0}{4}$ is a maximum. The area of such a rectangle is given by

$$A\left(\frac{P_0}{4}\right) = \frac{P_0}{4}\left(\frac{P_0}{2} - \frac{P_0}{4}\right).$$

2. (*5 points*) Find positive numbers x and y so that their product is 12 while the quantity $2x + y$ is as small as possible.

Solution: Let $L = 2x + y$. We need to minimize L with the condition that $xy = 12$, or equivalently, $y = \frac{12}{x}$. Hence

$$L(x) = 2x + y = 2x + \frac{12}{x}.$$

Now compute

$$L'(x) = 2 - \frac{12}{x^2}.$$

To find critical points we must solve $2 - \frac{12}{x^2} = 0$ which has solutions

$$x = \pm\sqrt{6}.$$

The problem only asked about positive numbers so we will only consider $x = \sqrt{6}$. We will determine extrema by the second derivative test. So compute

$$L''(x) = \frac{36}{x^3}.$$

Thus we can compute

$$L''(\sqrt{6}) = \frac{36}{(\sqrt{6})^3} > 0.$$

Hence by the second derivative test, the values $x = \sqrt{6}$ and $y = \dfrac{12}{\sqrt{6}}$ minimize L.