MATH 14 LAB SPRING 2014
Quiz 2 SOLUTIONS

Show all work clearly and in order, and circle your final answers.
Justify your answers algebraically whenever possible. Unjustified work may not receive full credit.
You have 25 minutes to complete this 10 point exam.

1. (*3 points*) Find the limit

$$\lim_{x\to 1} \frac{x^2-1}{x^2+x-2}$$

Solution: Notice if $x = 1$ is plugged into the expression, the result is $\frac{0}{0}$. So we must algebraically manipulate the function inside. Notice that

$$\frac{x^2-1}{x^2+x-2} = \frac{(x-1)(x+1)}{(x+2)(x-1)} = \frac{x+1}{x+2}.$$

Hence we compute

$$\begin{aligned} \lim_{x\to 1} \frac{x^2-1}{x^2+x-2} &= \lim_{x\to 1} \frac{x+1}{x+2} \\ &= \frac{1+1}{1+2} \\ &= \frac{2}{3}. \end{aligned}$$

2. (*3 points*) Find the limit

$$\lim_{x\to 1^+} \frac{2x^2+3x+7}{x-1}$$

Solution: Notice that plugging $x = 1$ into the expression yields $\frac{12}{0}$. Hence the limit will be infinite, but we must still determine whether it will be $+\infty$ or $-\infty$. Since we are approaching 1 from the right, consider a point x_0 slightly larger than 1. If we plug x_0 into the expresison $\frac{2x^2+3x+7}{x-1}$ we get a positive value, since the numerator is obviously positive and the denominator is also positive (since we defined $x_0 > 1$, we get $x_0 - 1 > 0$). Therefore we may conclude that

$$\lim_{x\to 1^+} \frac{2x^2+3x+7}{x-1} = +\infty.$$

3. (*4 points*) Find the limit

$$\lim_{t\to 4} \frac{t-\sqrt{3t+4}}{4-t}$$

Solution: If $t = 4$ is plugged into the expression, the result is $\frac{0}{0}$. So we will proceed by multiplication by conjugate. Compute

$$\begin{aligned}
\lim_{t\to 4}\frac{t-\sqrt{3t+4}}{4-t} &= \lim_{t\to 4}\frac{t-\sqrt{3t+4}}{4-t}\left(\frac{t+\sqrt{3t+4}}{t+\sqrt{3t+4}}\right)\\
&= \lim_{t\to 4}\frac{t^2-(3t+4)}{(4-t)(t+\sqrt{3t+4})}\\
&= \lim_{t\to 4}\frac{t^2-3t-4}{(4-t)(t+\sqrt{3t+4})}\\
&= \lim_{t\to 4}\frac{(t-4)(t+1)}{(4-t)(t+\sqrt{3t+4})}\\
&= -\lim_{t\to 4}\frac{(4-t)(t+1)}{(4-t)(t+\sqrt{3t+4})}\\
&= -\lim_{t\to 4}\frac{t+1}{t+\sqrt{3t+4}}\\
&= -\frac{4+1}{4+\sqrt{3\cdot 4+4}}\\
&= -\frac{5}{4+\sqrt{16}}\\
&= -\frac{5}{8}.
\end{aligned}$$