MATH 14-3D2 Spring 2014
Quiz 7 SOLUTIONS

Show all work clearly and in order (on this sheet or an attached sheet) and circle your final answers.
Justify your answers algebraically whenever possible. Work without justification may not receive credit.
You have 25 minutes to take this 10 point quiz.

Recall we can interpret differentiation as the the operator D that takes a differentiable function f to the function $D(f) = \frac{df}{dx}$. In this interpretation, the anti-derivative operator D^{-1} is defined so that it takes a function to a set of functions $D^{-1}(f)$ so that the derivative of all functions in the set $D^{-1}(f)$ is f.

1. (*3 points*) Compute the antiderivative of the polynomial

$$x^2 + 3x - 2.$$

Solution: $\frac{x^3}{3} + \frac{3}{2}x^2 - 2x + C$

2. (*3 points*) Compute $D^{-1}\left(\sin(x) + 2x^2 - \cos(x)\right)$.

Solution: $-\cos(x) + \frac{2}{3}x^3 - \sin(x) + C$

3. (*4 points*) Solve the initial value problem

$$\begin{cases} D(f) = \cos(x) - x^2 + 17 \\ f\left(\frac{\pi}{4}\right) = 1. \end{cases}$$

Solution: If we apply the D^{-1} operator to both sides we get

$$f(x) = D^{-1}\left[\cos(x) - x^2 + 17\right] = \sin(x) - \frac{x^3}{3} + 17x + C.$$

Our initial condition then implies

$$1 = f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) - \frac{\pi^3}{4^3 \cdot 3} + \frac{17\pi}{4} + C$$

and so

$$C = 1 - \frac{\sqrt{2}}{2} + \frac{\pi^3}{4^3 \cdot 3} - \frac{17\pi}{4}.$$

Therefore the solution of the initial value problem is

$$f(x) = \sin(x) - \frac{x^3}{3} + 17x + 1 - \frac{\sqrt{2}}{2} + \frac{\pi^3}{4^3 \cdot 3} - \frac{17\pi}{4}.$$