MATH 14-3D2 Spring 2014
Quiz 6 SOLUTIONS

Show all work clearly and in order (on this sheet or an attached sheet) and circle your final answers.
Justify your answers algebraically whenever possible. Work without justification may not receive credit.
You have 25 minutes to take this 10 point quiz.

1. (*3 points*) Find absolute extrema of the function $\begin{cases} f\colon [-2,3] \to \mathbb{R} \\ f(x) = x^2 - 10 \end{cases}$.

Solution: We see that $f'(x) = 2x$ and so the critical points of f solve the equation $2x = 0$. Hence the only critical point is at $x = 0$. We must now compare the values of f at the endpoints of the interval $[-2,3]$ and at 0:
$f(-2) = 4 - 10 = -6$
$f(3) = 9 - 10 = -1$
$f(0) = -10$
Therefore the absolute maximum of f occurs at $x = 3$ and the absolute minimum of f occurs at $x = 0$.

2. (*3 points*) Find the relative extrema of $\begin{cases} f\colon [-3,4] \to \mathbb{R} \\ f(x) = 2x^3 + 3x^2 - 12x + 1. \end{cases}$

Solution: We see that $f'(x) = 6x^2 + 6x - 12$ and so the critical points of f solve the equation $6x^2 + 6x - 12 = 0$. Hence the critical points of f are -2 and 1. We will proceed to check these points as relative extrema using the second derivative test.
Observe that $f''(x) = 12x + 6$. Now compute $f''(-2) = -24 + 6 < 0$ and conclude that a relative maximum of occurs at $x = -2$. Now compute $f''(1) = 12 + 6 > 0$ and conclude that f has a relative minimum at $x = 1$.

3. (*4 points*) Where is the function $\begin{cases} f\colon \mathbb{R} \to \mathbb{R} \\ f(x) = x^3 - 2x^2 + x \end{cases}$ concave up? concave down? Where are its inflection points?
Solution: Concavity depends on the second derivative, so observe that $f''(x) = 6x - 4$. We say that f is concave up if $f''(x) > 0$, so solve the inequality $6x - 4 > 0$ yielding solution $x > \dfrac{4}{6}$. Similarly, we may deduce

that $f''(x) < 0$ yields $x < \dfrac{4}{6}$ and $f''(x) = 0$ yields $x = \dfrac{4}{6}$.
Therefore f is concave up on the interval $(\frac{4}{6}, \infty)$, concave down on $(-\infty, \frac{4}{6})$, and has an inflection point at $\dfrac{4}{6}$.